KONINKLIJKE AKADEMIE VAN WETENSCHAPPEN
TE AMSTERDAM.

PROCEEDINGS

VOLUME XXV

Nos. 3 and 4.

President: Prof. F. A. F. C. WENT.

Secretary: Prof. L. BOLK.

(Translated from: "Verslag van de gewone vergaderingen der Wis- en Natuurkundige Afdeeling," Vol. XXXI).

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Mathematics. — *"Axes of Rotation of Quadratic Surfaces through 4 Given Points"*. By H. J. VAN VEEN. (Communicated by Prof. JAN DE VRIES).

(Communicated at the meeting of March 25, 1922).

§ 1. If we assume three points in space, any straight line l may be considered as the axis of rotation of a quadratic surface of revolution through these points. For the circles which the three points describe during the revolution round l, cut a plane through l in six points. These lie apparently on a conic k^2 which has l as axis of symmetry. Revolution of k^2 round l gives a quadratic surface of revolution (in what follows to be indicated by O^2), which has l as axis of rotation (briefly axis) and which passes through the 3 given points.

As a rule an O^2 is defined by its axis and three points; if, however, during the revolution round l two (or three) of the given points describe the same circle, there exists a *pencil* (*net*) of O^2's that have l for axis and pass through the 3 points.

An O^2 is always defined by 3 circles with the same axis, provided these circles do not all lie in the same plane.

§ 2. The axes of the O^2's through 4 given points $A_i\,(i=1\ldots4)$ form a complex of rayes Γ, which will be investigated in what follows. By O^2 we shall understand *a quadratic surface touching the sphere-circle* γ^2 *twice*; the line p joining the points of contact, will be called *chord of contact*; the conjugated polar line of p — defined as the locus of the points the polar planes of which pass through p — is *the axis of* O^2. As a rule this locus is a straight line p' passing through the pole P of p relative to γ^2; if p' is indefinite only the straight line (or lines) *conjugated to p and passing through* P will be considered as axis.

As special quadratic surfaces which according to the aforesaid must be considered as O^2, I mention: *a parabolical cylinder with* a plane pencil of axes in the plane $V\infty$ *at infinity* and *a pair of parallel planes* with a sheaf of // axes.

§ 3. Assume an arbitrary plane π and in it a point P. If Q is the point at infinity of a straight line of the plane pencil (P,π), PQ can only be the axis of an O^2 touching γ^2 in its points of intersection with the polar line q of Q relative to γ^2, but at the same time the polar plane of P relative to the same O^2, must pass through q.

The O^2's through A_i touching γ^2 at its points of intersection with q, form a pencil; if Q moves along the straight line at infinity r of π, q revolves round the pole R of r relative to γ^2. We get in this way ∞^2 O^2's cutting V_∞ in a system of ∞^2 conics k^2 touching γ^2 at its intersections with a ray of the plane pencil round R.

Now I represent the space of the conics of V_∞ on a five dimensional point-space R_5 by considering the coefficients of the equation of a k^2 as the homogeneous coordinates of a point in R_5; to a conic k^2 and to a linear system of ∞^k conics $(k^2)_k$ of V_∞ there correspond a point and a linear space R_k of R_5 and inversely.

The double straight lines of a $(k^2)_4$ of V_∞ envelop a conic; two of those double lines pass through R, hence the image of all double lines through R has 2 points in common with an arbitrary R_4; it is a conic k_1^2. To γ^2 there corresponds a point P and to the pencils touching γ^2 in its points of intersection with rays of the plane pencil round R, there correspond the generatrices of the cone K that has P as vertex and k_1^2 as directrix.

All the quadratic surfaces through A_i relative to which P and one of the straight lines q are harmonically conjugated, form a linear system of ∞^3 individuals, an $(O^2)_3$; this cuts V_∞ in a $(k^2)_3$ to which there corresponds an R_3 in R_5. Considering the quadratic surfaces through A_i relative to which P and R and P and V_∞ are conjugated, it appears that the R_3's corresponding to *all* the straight lines q, pass through an R_2 and lie in an R_4. These R_3's cut the space R_3 in which K lies, in a plane pencil the rays of which by means of the straight lines q are projectively associated to the generatrices of K. It happens three times that the associated elements coincide, hence there exist three O^2's through A_i that have a straight line q as chord of contact and the polar plane of P relative to such an O^2 passes through q. To a plane pencil (P,π) there belong therefore three rays of Γ or:

the complex Γ of the axes of rotation of the quadratic surfaces of revolution through 4 *given points is of the order* 3, *the complex cones are of the order three, the complex curves of the class three.*

§ 4. Algebraically the order of Γ may be found by determining e.g. the complex cone of an arbitrary point. With a view to this I

take this point as the origin of a rectangular system of coordinates. The equation of an arbitrary quadratic surface of revolution is:

$$f(x,y,z) \equiv x^2 + y^2 + z^2 + \alpha(ax+by+cz)^2 + 2Ax + 2By + 2Cz + D = 0.$$

The axis of revolution is defined by the equations:

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = \frac{\partial f}{\partial z}$$
$$a \qquad b \qquad c$$

or

$$\frac{x+A}{a} = \frac{y+B}{b} = \frac{z+C}{c}$$

and passes through O if

$$\frac{A}{a} = \frac{B}{b} = \frac{C}{c} = \beta.$$

Consequently only the axes of the surfaces

$$x^2 + y^2 + z^2 + \alpha(ax + by + cz)^2 + 2\beta(ax + by + cz) + \gamma = 0$$

pass through O.

We only consider O^2's through the four given points (x_i, y_i, z_i), hence:

$$x_i^2 + y_i^2 + z_i^2 + \alpha(ax_i + by_i + cz_i)^2 + 2\beta(ax_i + by_i + cz_i) + \gamma = 0;$$

elimination of α, β and γ gives:

$$\begin{vmatrix} x_i^2 + y_i^2 + z_i^2 & (ax_i + by_i + cz_i)^2 & ax_i + by_i + cz_i & 1 \end{vmatrix} = 0$$

As a, b, c are the direction cosines of an axis through O, they are proportional to the coordinates of an arbitrary point of such a straight line. Consequently the equation of the complex cone of O becomes:

$$\begin{vmatrix} x_i^2 + y_i^2 + z_i^2 & (xx_i + yy_i + zz_i)^2 & xx_i + yy_i + zz_i & 1 \end{vmatrix} = 0.$$

In a similar way an equation may be derived defining the rays of Γ in an arbitrary plane.

§ 5. If the origin of a rectangular system is placed at the centre of the sphere through A_i, the equation of the complex of rays in line coordinates may be written:

$$\begin{vmatrix} 0 & p_4 & p_5 & p_6 & 0 \\ P_1^2 & x_1 & y_1 & z_1 & 1 \\ P_2^2 & x_2 & y_2 & z_2 & 1 \\ P_3^2 & x_3 & y_3 & z_3 & 1 \\ P_4^2 & x_4 & y_4 & z_4 & 1 \end{vmatrix} = 0$$

where $P_i = p_1 x_i + p_2 y_i + p_3 z_i$.

§ 6. All the straight lines through the centre M of the sphere B passing through the points A_i, are rays of Γ. Likewise all the straight lines perpendicular to a side plane of the tetrahedron T that has A_i as angular points; for they are axes of the O^2 consisting of that plane and a parallel plane through the 4th angular point. Further any line perpendicular to 2 subtending sides of T belongs to Γ; they are the axes of the O^2 consisting of the pair of parallel planes through these 2 sides, hence:

the complex Γ has 8 cardinal points: M, the points D_i at infinity of the normals to the side planes and the points at infinity H_j of the normals to the subtending sides of T.

§ 7. If the points A_i revolve round a straight line l, lying in a perpendicular bisector plane of a side of T, 2 of the points A_i describe the same circle; from this follows that l belongs to Γ, or:

the six perpendicular bisector planes of the sides of T are cardinal planes of Γ.

I shall now show, that all the straight lines of V_∞ are double rays of Γ.

§ 8. The axes corresponding to an arbitrary point P of V_∞ belong to a pencil (O^3); they are the straight lines p' conjugated to the polar line p of P relative to γ^2. The centres of the individuals of the pencil lie on the polar line p of P relative to γ^2 (they belong to the parabolical cylinder of the pencil) and on a conic passing through P and M and intersecting p. The axes through P form consequently a plane pencil in V_∞ and a pencil the plane of which passes through M, hence:

the complex Γ consists of ∞^3 plane pencils of parallel rays lying in the planes of the sheaf round M.

From this there follows that Γ is invariant for any homothetic transformation relative to M; the complex cones corresponding to the points of a straight line through M, have accordingly the same curve at infinity.

§ 9. All the straight lines of V_∞ belong to Γ, hence the complex curve of an arbitrary plane π touches the l_∞ of its plane. Besides this straight line one more tangent may be drawn to the complex curve out of each point P of this l_∞, namely the line of intersection of π with the plane of the pencil of complex rays through P passing through M. Consequently l_∞ is a bi-tangent of the complex curve of π and also of all the planes in which it lies, or:

V_∞ carries a field of double rays of Γ.

§ 10. *The complex curve of an arbitrary plane π is rational; the l_∞ of its plane is its bi-tangent; single tangents are: the lines of intersection of π with the perpendicular bisector planes of T.*

Through its bi-tangent and the six single tangents the complex curve of an arbitrary plane is defined; other tangents may be constructed with the ruler.

§ 11. If the tetrahedon T is cut by V_∞, we get the well known configuration of a complete quadrilateral. Polarisation of this figure in the absolute polar field gives a complete quadrangle having D as angular points; the straight lines at infinity of the perpendicular bisector planes are the sides and the points H_j are the diagonal points of this quadrangle.

§ 12. In a plane π through one of the points H_j, hence parallel to a normal to 2 subtending sides of T, the complex rays consist of *a plane pencil* round H_j and the tangents of *a parabola*. If π passes at the same time through M, it contains also a plane pencil round M, hence also a third plane pencil; as the l_∞ of π is a double ray of Γ, the centre of this third plane pencil lies also on l_∞.

In a plane π through 2 of the points H_j there lie plane pencils round both these points, hence also a third plane pencil; to this belong the points of intersection of π with the perpendicular bisector planes through the third of the points H_j, hence:

to Γ there belong three bilinear congruences, which have as directrices the join of 2 of the points H_j and the line through the 3rd of the points H_j and M.

If π passes through 2 points H_j and through M, the complex rays in π form the plane pencils round these three points.

In a plane π through one point H_j and two of the points D_i there lie three plane pencils of complex rays round these points. If π passes also through M it is a cardinal plane.

§ 13. Before investigating the planes through a point D_i I shall first consider the complex cone of a point P of the perpendicular m_i out of M to one of the side planes of T. This complex cone is apparently split up into three plane pencils, lying in the perpendicular bisector planes through m_i; m_i is a threefold generatrix of the complex cone of each of its points, hence:

the four straight lines m_i are 3 fold rays of Γ.

In a plane π through m_i lies a plane pencil round M and a plane pencil round D_i; now the l_∞ of π is a double ray and m_i is a

threefold ray of Γ, hence the third plane pencil in π has likewise D_i as vertex; the complex rays in π form accordingly a plane pencil round M and a plane pencil round D_i which is to be counted double.

§ 14. Consider an arbitrary plane π through one of the points D_i; in this there lies *a plane pencil* of complex rays round D_i, while the rest of the rays envelop *a parabola*. Out of each point P of the l_∞ of π there can be drawn besides l_∞ one more tangent to the parabola; P is the point of contact if this straight line coincides with l_∞. The plane of the pencil of complex rays through P passes in this case through M and through D_i, hence through m_i, but then P coincides with D_i or:

in a plane through one of the points D_i ($/\!/$ to a straight line m_i) the complex rays consist of a plane pencil round this point and of the tangents of a parabola with axis $/\!/ m_i$.

§ 15. In a plane through M there lies a plane pencil of rays round this point and as the l_∞ of this plane p is a double ray of Γ there lie 2 more plane pencils with centres P on p. The points P and the straight lines p are conjugated in a null system [2,1]. By conjugating to each other the points P lying on the same straight line, an involution of pairs [2] arises. This involution is quadratic, for on an arbitrary straight line there lies one pair of conjugated points.

The involution [2] is not a quadratic inversion as the joins of conjugated points do not pass through a fixed point; consequently [2] consists of the pairs of points conjugated to each other relative to the conics of a pencil. This involution has 4 *double points* (the base points of the pencil), in this case the points D_i, and 3 *cardinal points*, the diagonal points of the complete quadrangle of the base points, in our case the points H_i, hence:

the complex Γ consists of pairs of plane pencils of parallel rays lying in planes through M. The vertices of the two plane pencils lying in the same plane, are conjugated points of a quadratic involution in V_∞.

§ 16. If a straight line p of V_∞ revolves round one of its points O, the points associated to p in the null system [2,1] describe a curve k^3 of the 3rd order; this curve passes through O, through H_j and touches the straight lines OD_i at D_i. The curves k^3 belonging to all the plane pencils of V_∞, form a net with seven base points, H_j and D_i.

§ 17. In order to get the complex cone of an arbitrary point P, we consider a plane π through MP; let O be the intersection of MP, p the intersection of π with V_∞. If P_1 and P_2 correspond to p, PO, PP_1 and PP_2 are the lines of intersection of the complex cone of P with π. If π revolves round PO it appears that:

the complex cone of a point P passes through the straight lines PM and PH_j and touches the planes MPD_i along the lines PD_i.

At the same time it appears again that if P moves along a straight line through M, the curve at infinity of the complex cone of P remains unaltered (cf. § 8).

§ 18. Out of a point O, 4 real tangents OD_i may be drawn to the corresponding curve k^3, hence the curves k^3 and also *the complex cones consist of two parts.*

The caracteristic of a curve k^3 is defined by the 4 straight lines OD_i. Through D_i there pass 3 conics through the points O of which there pass 4 harmonical rays through D_i, hence:

the locus of the points with harmonical complex cones consists of 3 *quadratic cones the vertices of which lie in M and which pass through the straight lines m_i,* and also:

the complex cones of the points lying on a quadratic cone through the 4 *straight lines m_i, have the same characteristic.*

§ 19. The curve of JACOBI of the net of the curves k^3 consists of the six sides of the complete quadrangle of the points D_i. No rational curves k^3 belong to the net, only curves degenerated in a side of the quadrangle and a conic through the 4 points D_i and H_j that do not lie on this side, accordingly:

there are no points with rational complex cones; for any point of a perpendicular bisector plane the complex cone degenerates into a plane pencil and a quadratic cone; for a point V_∞ the complex cone consists of a plane pencil in V_∞, to be counted double, and a single plane pencil.

§ 20. As each complex curve has a double tangent, we might call those planes where the double tangent is an inflexional tangent, *singular* planes. In this case the two points P corresponding to the straight line p in the null system [2,1], must coincide. This happens only when a plane π passes through one of the points D_i, but then the system of complex rays in π splits up into a plane pencil and the tangents of a parabola; *consequently non-degenerate complex curves with an inflexional tangent do not occur.*

§ 21. If in π there lies a plane pencil with centre P at finite distance, there are also 2 plane pencils with their centres on the l_∞ of π; if π does not pass through one of the cardinal points at infinity, the planes of these latter pencils pass through M, hence:

only the planes through the 8 *cardinal points contain degenerate complex curves* (cf. §§ 12, 13 and 14).

§ 22. As the null system [2,1] and the involution [2] are invariant for central projection, we can *construct the complex cone* of an arbitrary point P in the following way:

We determine the points of intersection D_i of the perpendiculars out of P to the side planes of T with an arbitrary image plane τ and also the intersection O of TM with τ. Then we construct the double points of the quadratic involution in which the conics of the pencil through D_i cut an arbitrary straight line l through O; we fix this involution by means of the points of intersection of l with 2 degenerate conics of this pencil. The straight lines joining P to the double points in question, are generatrices of the complex cone of P.

§ 23. If the points A_i are coplanar, their plane α cuts an O^2 of the system in consideration along a conic k^2 through A_i or is a part of the O^2. In the first case the axis of O^2 lies in one of the planes through the axes of symmetry of k^2 perpendicular to α; in the second case the axis of O^2 is a straight line perpendicular to α. The axes of symmetry of the conics through A_i are tangents to a curve of the 3rd class touching the line l_∞ of α twice; the planes through these axes and $\perp \alpha$ touch a cylinder of the 3rd class with V_∞ as double tangent plane.

The rays of Γ in an arbitrary plane π touch also in this case at a curve of the 3rd class that has the l_∞ of its plane as a bitangent. The complex cone of an arbitrary point P, however, splits up into 3 plane pencils the planes of which touch at the cylinder in question; a perpendicular to α is a triple generatrix of the complex cone of each of its points, hence:

if the 4 *points* A_i *are coplanar,* Γ *consists of the tangents to a cylinder of the* 3rd *class;* V_∞ *is the bearer of a field of double rays; the vertex of the cylinder at infinity is the bearer of a sheaf of triple rays.*

§ 24. Now consider the case that 3 of the points A_i lie on a straight line a; then the O^2's must pass through a fixed point A

and a fixed straight line a. If A is to lie on the quadratic surface which a describes when it revolves round a straight line l, the circle which A describes when it revolves round l, must cut the straight line a; accordingly l must lie in the plane which bisects perpendicularly the straight line joining A to a point of a. These planes touch a parabolical cylinder that has for directrix the parabola of which A is the focus and a the director line and the generatrices of which are perpendicular to the plane (A,a).

If a revolves round a straight line crossing it at right angles, a plane is produced which, completed by the plane through A parallel to it, gives another O^2 that satisfies the conditions mentioned, hence:

if three of the four points A_i are collinear, Γ splits up into a pencil of rays with the axis at infinity, and the tangents to a parabolical cylinder.

Mathematics. — *"Axes of Rotation and Planes of Symmetry of Quadratic Surfaces of Revolution through 5, 6 and 7 Given Points."* By H. J. VAN VEEN. (Communicated by Prof. JAN DE VRIES).

(Communicated at the meeting of April 29, 1922).

§ 1. Let there be given five points A_1 A_2 B_j $(j = 1, 2, 3)$. I consider the complexes Γ_1 and Γ_2 belonging to the points A_1 B_j and A_2 B_j [1]). Generally a common ray l of Γ_1 and Γ_2 is the axis of an O^2 through the 5 points; for l is the axis of an O^2 through A_1 B_j and of an O^2 through A_2 B_j; these two O^2's have in common the 3 parallel circles on which the B_j lie; hence they coincide. An exception exists for the straight lines in the perpendicular bisector plane of the join of 2 of the points B_j, and also for the straight lines of V_∞. The field degree of the congruence of axes is therefore

$$3.3 - 3 - 2.2 = 2.$$

At the same time there must be split off: the sheaf of the rays which are perpendicular to the plane through the points B_j. Let D be the centre at infinity of this sheaf; both the complex cones of a point P touch the plane through P M_1 M_2 and D along P D, hence:

the axes of the O^2's through 5 given points form a congruence of the sheaf degree 7 and the field degree 2, $C^{7,2}$.

§ 2. To $C^{7,2}$ belong the complex rays of Γ_2 lying in the perpendicular bisector plane of a straight line A_1 B_j, hence:

the 10 perpendicular bisector planes of the joins of the 5 given points are singular planes of the order 3.

§ 3. In the two null systems belonging to Γ_1 and Γ_2 the curves k_1^3 and k_2^3 (O^2's through 4 points § 16) are associated to a plane pencil round a point O of V_∞; these curves pass through O, touch O D at D and have accordingly six more points in common. Consequently through O there pass six straight lines on which the two pairs of points which through the two null systems are associated to them, have one point in common.

[1]) Cf. my paper "Axes of Rotation of Quadratic Surfaces through 4 Given Points."

The complex curves in an arbitrary plane through such a straight line touch each other at the point in question, so that the two complex curves have 5 coinciding tangents in common in the l_∞ of their plane. Now we have split off the straight lines of V_∞ as 4-fold rays of the congruence of the intersection of the two complexes, hence:

V_∞ is a singular plane of the order 6.

§ 4. We can also arrive at this last result in the following way. The quadratic surfaces through 5 points form a linear system of ∞^4 individuals; these cut V_∞ in a $(k^2)_4$; the conic of the double straight lines of this $(k^2)_4$ belongs to the parabolical cylinders of $(O^2)_4$. Let C be such a cylinder, T its vertex, c the line along which C touches V_∞.

The polar plane of T relative to C is indefinite, hence T has a fixed polar plane relative to all O^2's of the pencil through $A_1 B_j$, which touch γ^2 in its points of intersection with c. This fixed polar plane is at the same time the plane of the centres of the individuals of the pencil; it passes through the polar line p of T relative to γ^2. In the null system $[2,1]$ belonging to Γ_1 the pole P of c relative to γ^2 is associated to p.

As the fixed polar plane of T relative to the O^2's through $A_2 B_j$ that touch γ^2 at its points of intersection with c, pass likewise through p, in the two null systems corresponding to Γ_1 and Γ_2 the point P is associated to p.

P was the pole of c relative to γ^2, hence the locus of P is a conic. The order of the null systems is three; accordingly the locus of the straight line p *is a curve of the sixth class.*

We remark also that to each parabolical cylinder one axis in V_∞ remains associated (O^2's through 4 points, § 2), namely the polar line of its vertex relative to γ^2.

§ 5. If *six* points are given I consider a group of 4 and a group of 5 of these points which have 3 points in common. To the group of 4 points there belongs a complex Γ^3, to that of 5 points a congruence $C^{7,2}$. The axes in question are part of the common rays of complex and congruence; however, we must split off: the tangents of three curves of the 3rd class and twice the tangents of a curve of the sixth class, so that we arrive at a ruled surface of the order $3(7+2) - 3.3 - 2.6 = 6$, hence:

the axes of the O^2's through six points form a ruled surface of the sixth order, ϱ^6.

§ 6. Through consideration of the perpendicular bisector plane of the straight line through the 2 points that belong to the group of 5 and not to the group of 4 points, we find that in this plane and accordingly in each of the 15 perpendicular bisector planes, there lie 2 generatrices of ϱ^6.

The quadratic surfaces through six points cut V_∞ in a linear system of ∞^1 conics $(k^2)_3$. These define together with γ^2 a linear system $(k^2)_4$; the tangents of the conic of the double lines of $(k^2)_4$ are the chords of contact of the O^2's through the six points; polarisation of these straight lines relative to γ^2 gives a conic k^2; to $(k^2)_3$ there belong four double lines, originating from parabolical cylinders (cf. § 4), so that the locus of the axes has a conic k^2 and 4 straight lines in common with V_∞, hence:

ϱ^6 *is rational; it has a double curve of the order* 10; *the* 15 *perpendicular bisector planes of the joins of the six points are bi-tangent planes;* V_∞ *is a* 4*-fold tangent plane.*

§ 7. In order to investigate the axes of the O^2's through *seven* points, we consider a group of 4 and a group of 6 of these points that have 3 points in common. We get in this way a complex Γ_3 and a ruled surface ϱ^6 that have 18 straight lines in common. If we subtract from them three times two straight lines lying in the perpendicular bisector planes of the joins of the 3 common points, and twice 4 straight lines in V_∞, we have 4 straight lines left, hence:

through 7 *points there pass* 4 O^2's.

§ 8. We can also arrive at this result in the following way. All quadratic surfaces through 7 points cut V_∞ in a $(k^2)_2$; in connection with γ^2 this gives a $(k^2)_3$ with 4 double straight lines, consequently in $(k^2)_3$ there are *four* individuals touching γ^2 twice. These belong to the surfaces of rotation through the 7 points.

§ 9. A quadratic surface of revolution O^2 has a pencil of planes of symmetry passing through the axis of rotation and therefore defined together with this axis, and further generally one more plane of symmetry perpendicular to the axis. I shall investigate these latter planes for O^2's through given points and I define as *a plane of symmetry* of an O^2 the polar plane of the point P at infinity of the axis of rotation; if this polar plane is indefinite the planes through the chord of contact p of the O^2 are considered as planes of symmetry.

§ 10. An arbitrary plane π is a plane of symmetry of one O^2 through *four* given points A_i; for through A_i there passes a pencil of O^2's touching γ^2 at its points of intersection with π; generally one of these O^2's passes through the mirror image of one of the points A_i relative to π and this surface satisfies the conditions.

It may happen that the mirror image in question lies on the base curve of the pencil; then π is a plane of symmetry of all individuals of the pencil. As the sphere B through A_i belongs to the pencil, π must pass in this case through the centre M of this sphere.

§ 11. The ∞^2 planes of symmetry of the O^2's through *five* points envelop a surface of which I shall determine the class. The O^2's the planes of symmetry of which pass through a point P of V_∞, cut V_∞ along conics that touch γ^2 at its points of intersection with a ray of the plane pencil round P. The image of all such conics in R_5 is a quadratic cone K (O^2's through 4 points § 3).

The quadratic surfaces through the 5 given points cut into V_∞ a $(k^3)_4$ that has an R_4 as image in R_5; this R_4 cuts K along a conic $k^3{}_{II}$.

To the degenerate conics of V_∞ there corresponds in R_5 *a cubic hypersurface*, $V^3{}_4$, that has *a double surface* $O^4{}_2$ *of the* 4^{th} *order* (a surface of *Veronese*). Besides its two points of intersection with $k^3{}_I$ (O^2's through 4 points, § 3) that lie on $O^4{}_2$, $k^3{}_{II}$ has 2 more points in common with $V^3{}_4$, hence to the O^2's through the 5 given points the planes of symmetry of which pass through P, there belong *two paraboloids of revolution*; these have V_∞ as a plane of symmetry. Through a ray p of the plane pencil round P there passes one more plane of symmetry that does not coincide with V_∞, consequently the planes of symmetry through P envelop a cone that has P for vertex and that touches V_∞, twice. An arbitrary straight line l through P bears therefore 3 planes of symmetry; through a line of V_∞ there passes besides V_∞ only one more plane of symmetry, hence:

the planes of symmetry of the O^2*'s through* 5 *given points envelop a surface of the* 3^{rd} *class of which* V_∞ *is a double-tangent plane.*

§ 12. The conic along which this surface touches V_∞, has six tangents that are the bearers of pencils of tangent planes; these cannot belong to different O^2's for in that case through the 5 given points there would pass a pencil of O^2's touching γ^2 at its points of intersection with a straight line p and from this would follow that the 5 given points must lie on a sphere.

To each of the six straight lines p belongs therefore one O^3 that has a pencil of parallel planes of symmetry, or:

through 5 *given points there pass six cylinders of revolution; their generatrices are parallel to* 6 *sides of a quadratic cone.*

§ 13. The planes of symmetry through an arbitrary point touch a cone of the 3rd class; let π be such a plane through the centre M of the sphere B through 4 of the 5 given points; π is then a plane of symmetry of an O^3 through the 5 points and also of the sphere B, hence of a pencil of O^3's through those 4 points, or:

through the centre of the sphere through 4 *given points there pass* ∞^1 *planes each of which is a plane of symmetry of a pencil of* O^3*'s through those* 4 *points; these planes envelop a cone of the* 3rd *class.*

Such a plane π is also a plane of symmetry of the base curve of the corresponding pencil, consequently to this pencil there belongs a cylinder of revolution of which the generatrices are perpendicular to π, hence:

through 4 *points there pass* ∞^1 *cylinders of revolution of which the generatrices are parallel to the generatrices of a cone of the* 3rd *order.*

§ 14. If *six* points are given, we consider two groups of five points; these have 4 points in common. The surfaces of the 3rd class corresponding to these two groups, have in common the tangent planes of a developable surface of the 9th class that has V_∞ as a 4-fold tangent plane. However, we must subtract from this the tangent planes through the centre of the sphere through the 4 common points, hence:

the planes of symmetry of the O^3*'s through six given points envelop a developable surface of the* 6th *class that has* V_∞ *as a* 4*-fold tangent plane.*

§ 15. The quadratic surfaces through six points cut V_∞ in a $(k^3)_3$; to this there belong 4 double straight lines; how many degenerate curves touching γ^2 twice, belong to $(k^3)_3$?

In order to determine this number we remark that the cone in R_6 formed by the straight lines joining the image of γ^2 to O^4_2 (§ 11), cuts the image R_3 of $(k^3)_3$ along a curve k^4 of the 4th order; this curve has besides the 4 points that are the images of the double lines of $(k^3)_3$ and that are to be counted twice, 4 more points in common with V^3_4, hence:

through six points there pass 4 *parabolical cylinders and* 4 *paraboloids of revolution.*

As through an arbitrary point P of V_∞ there pass 2 more planes of symmetry, (for the conic of the double lines of $(k^2)_4$ defined by γ^2 and $(k^2)_3$ sends two of them through P), we find also in this way, that the planes of symmetry in consideration envelop a developable surface of the sixth class with V_∞ as a 4-fold tangent plane.

§ 16. In order to find the planes of symmetry through *seven* given points, we consider a group of six and a group of five of these points that have 4 points in common. The corresponding surfaces have $3 . 6 = 18$ tangent planes in common. If we subtract from them $2 . 4 = 8$ times V_∞ and further 6 planes through the centre of the sphere through the 4 common points, it appears again that:

through seven given points there pass 4 *quadratic surfaces of revolution.*

(Cf. §§ 7 and 8).

Physics — *"A Connection between the Spectra of Ionized Potassium and Argon."* (First Communication.) By Prof. P. ZEEMAN and H. W. J. DIK.

(Communicated at the meeting of April 29, 1922).

According to the conception of RUTHERFORD-BOHR an atom consists of a very small positively charged nucleus, which contains almost the whole mass of the atom, and of a number of electrons revolving round the nucleus. The number of electrons moving round the nucleus, is equal to the atomic number of the element; hence this also indicates the number of units of charge which an atom that is neutral taken as a whole, possesses in the nucleus.

It has been made plausible that the electrons are arranged in shells or sheaths with the nucleus as centre. In particular the regular changes which the chemical properties undergo with the increase of the atomic number, make this probable. Regularly elements occur in the periodic system which easily cede one electron (the alkalis), regular is also the succession of the inert gases. This becomes comprehensible when it is assumed that a shell can become full, and that then the configuration will be very stable: helium, neon, argon etc. The atoms of lithium, sodium, potassium etc. have only one electron in the outer shell. On this similarity in structure rests also the resemblance which has been observed at an early date in the arc-spectra of the alkalis. The one outer electron can be removed by the electric forces which are active in a spark. Then the atom is ionized, and the electron combination which has remained, can emit the spark spectrum.

On these general features of the atomic model, in particular on the number of outer electrons which increases at every step in the periodic system, rests a displacement law enunciated by KOSSEL and SOMMERFELD[1]), which establishes a connection between the spark spectrum of an element and the arc-spectrum of another element which precedes it in the periodic system. If e.g. an electron of the potassium-atom has been driven out, the remaining electron system must present the greatest resemblance with that of argon, and only

[1]) KOSSEL u. SOMMERFELD, Auswahlprincip und Verschiebungssatz bei Serienspectren. Verh deutsch. physik. Gesellsch. 21. Jahrgang 240, 1919.

differ from it in that the positive nucleus of potassium possesses one unit of charge more. Like the arc-spectrum of argon, the spark-spectrum of potassium must be composed of a great number of lines, and not show series. As yet the relation which the displacement law renders probable, is only qualitatively known.

For some time some researches have been in progress in the Amsterdam laboratory to determine the relation quantitatively.

We will here communicate some results to which the investigation of *potassium* has led. These facts retain their value whatever interpretation may have to be given to them.

Besides the arc-spectrum of potassium with the so well-known spectrum series which according to SOMMERFELD's opinion originates from the neutral atom, EDER and VALENTA [1]) observed in 1894 a spectrum, emitted by ionized potassium, which was very rich in lines. EDER and VALENTA observed simultaneously arc- and spark lines; in 1907 GOLDSTEIN [2]) however, succeeded in observing in the intensely luminous line of discharge occurring in the passage of vigorous electric discharges through powdered salts, a spectrum in which only lines are seen which have not been ranged into series, and in which even the distinct arc-lines did not appear. GOLDSTEIN points out that these lines owe their origin to circumstances which differ essentially from those which give rise to the arc-lines, and he introduces the name of "ground" spectrum. We are undoubtedly justified in attributing the ground spectra to the emission of the once ionized atom.

In the subjoined Table I a * denotes the strongest lines, those published by GOLDSTEIN.

With better appliances EDER's pupil SCHILLINGER [3]) could supplement GOLDSTEIN's observations by investigating also the ultra-violet. He worked with vigorous discharges between potassium electrodes in a bulb with hydrogen. His observations are given under S in Table I.

In 1915 some observations of NELTHORPE [4]) were published for potassium lying between 6307 Å and 3898 Å. He employed another type of tube than GOLDSTEIN, and recorded by means of a spectrograph. On his plates the arc-lines are absent, the ground-spectrum of GOLDSTEIN coinciding with his strongest lines. The doubt expressed by KAYSER whether GOLDSTEIN's failure to see the

[1]) EDER u. VALENTA, Denkschriften Wien. Akad. **61**. 347, 1894.
[2]) GOLDSTEIN, Verh. deutsch. physik. Ges. 321. 1907; 426, 1910.
[3]) SCHILLINGER, Wiener Sitz. Ber. **118** [2*a*] 605. 1909.
[4]) NELTHORPE. Astroph. Journ. **41**. 16. 1909.

TABLE I. Potassium lines with electrode less discharge.

Intensity.					λ	ν	Remarks
EV	S	N	McL	D			
8	5				7699.4		Arc-line
8	5				7665.6		»
8	6			—	6938.9		»
7	4			—	6911.3		»
—	—		—	1	± 6594	± 15165	P_1
—	—		—	2	± 6562		H 6563?
—	1	—		1	6427.9	15557	
—	2	3		8	6307.2 *	15855	
—	1	1		5	6246.5 *	16009	Q_1
—	2	8		15	6120.2 *	16340	P_2
—	—	—	3	—	6050		
3	4	—		—	5832.2		Arc-line
3	2	—		—	5812.5		»
4	5	—		—	5802.0		»
3	3	—		—	5782.6		»
—	—	—	—	1	± 5772	17324	
—	—	—	2	—	5730		
—	—	—	—	3	± 5645	17715	S_1
—	—	—	—	1	± 5536	18064	R_2
—	—	—	—	4	5469	18286	P_3
8	2	—		—	5359.9		Arc-line
1	1	—		—	5343.3		»
8	2	—		—	5340.0		»
2	1	—		—	5323.6		»
—	—	—	—	3	5314	18817	
2	1	—		—	5112.7		Arc-line
2	1	—		—	5099.6		»
1	1	—		—	5085.1		»
1	1	—		8	5056.0	19778	P_4

TABLE I (*Continued*).

Intensity.					λ	ν	Remarks.
EV	S	N	McL	D			
2	2	5		15	5005.5 *	19978	R_3
1	—	—	—		4965.5		Arc-line
—	—	—	—		4958		»
1	1	—		5	4943.2	20230	P_5
—	—	—	3	—	4863		Arc-line
3	2	12		30	4829.2 *	20707	
—	—	—	1	—	4805		Arc-line
—	—	—	1	—	4790		»
—	—	—	1	—	4769		
—	—	—	1	—	4760		»
—	—	—	1	3	4744	21079	Q_5
—	—	—	1	—	4720		
—	—	—	1	—	4688		
3	1	—		15	4659.8	21460	R_4
2	—	—	—	—	4650.7		
—	—	—	1	—	4643		Arc-line
6	4	15		30	4608.5 *	21699	
—	—	—	—	10	4596.0	21758	P_6
5	4	8		30	4505.6 *	22195	P_7
5	2	5		20	4467.5 *	22384	P_8
1	—	—	2	9	4455.5	22444	P_9
1	1	1		10	4423.6	22606	Q_6
3	3	10		30	4388.3 *	22788	
—	—	—	2	10	4365.1	22909	P_{12}
—	1	5		30	4339.9	23042	Q_7
1	5	?		30	4309.5 *	23204	P_{13}
2	3	4		30	4305.0 *	23229	Q_8
—	1	—		9	4288.6	23317	
—	—	—	—	8	4285.1	23337	Q_{10}

TABLE I (*Continued*).

Intensity.					λ	ν	Remarks.
EV	S	N	McL	D			
6	8	10		30	4263.5 *	23455	R_6
6	4	6		30	4225.7 *	23665	Q_{11}
6	5	8		30	4223.2	23679	
1	1	—		9	4208.9	23759	Q_{12}
8	10	20		30	4186.2 *	23888	R_7
6	5	10		30	4149 1 *	24102	P_{14}
6	5	10		30	4134.7 *	24185	R_{10}
4	3	10		30	4115.1 *	24301	S_6
—	—	5	—	—	4106.8		
—	—	7	1	--	4104.2		
—	—	3	—	-	4098.6		
—	1	2		15	4093.5	24429	P_{15}
—	—	2	—	—	4086.8		
—	—	3	—	—	4075.6		
—	—	2	—	—	4072.3		
—	1	1		10	4065.2	24599	R_{12}
—	2	7		—	4058.1		
10	10	—		—	4047.4		Arc-line
10	20	—		—	4044.3		Arc-line
—	2	5		15	4042.5	24737	$P_{17}\,S_7$
1	1	—		10	4039.9	24753	
1	1	—		10	4024.9	24845	P_{18}
1	1	4		9	4018.9	24882	P_{19}
2	1	1		10	4012.2	24924	S_8
6	5	8		15	4001.1	24993	$P_{20}\,S_9$
1	4	3		10	3995.0	25031	S_{10}
—	—	—		8	3992.0	25050	P_{21}
3	3	4		15	3972.8	25171	P_{22}
	3	3		15	3966 7	25210	P_{23}

TABLE I (*Continued*).

Intensity.					λ	ν	Remarks.
EV	S	N	McL	D			
4	4	8		15	3955.5	25281	Q_{15}
2	1	4		10	3943.3	25359	S_{11}
1	1	1		9	3934.6	25416	
1	1	2		9	3927.3	25463	S_{12}
1	1	—		9	3923.8	25485	P_{24}
8	8	10		15	3898.0	25654	
—	—		—	3	3887.2	25726	Q_{19}
1	—			5	3884.5	25743	S_{13}
1	1			8	3879.2	25779	P_{25}
2	1			10	3874.5	25810	R_{14}
1	1			10	3861.9	25894	Q_{21}
—	—		—	3	3844.8	26009	P_{26}
1	2			15	3818.6	26187	P_{27}
—	1			—	3816.9		
1	2			15	3800.8	26310	R_{16}
3	3			15	3783.2	26433	R_{17}
1	3			15	3767 1	26546	R_{18}
1	1			6	3756.0	26624	Q_{25}
1	—		—	—	3749.1		
1	1			9	3745.2	26701	R_{20}
1	1			9	3739.2	26744	R_{21}
1	—		—	—	3727 5		
—	1			9	3722.1	26866	R_{22}
1	1			9	3716.9	26904	R_{23}
1	—		—	—	3713.2		
—	1			—	3683.7		
4	4			15	3682.3	27157	S_{16}
—	1			8	3677.6	27192	P_{29} R_{24}
1	1			10	3670.2	27246	P_{30}

TABLE I (*Continued*).

Intensity.					λ	ν	Remarks.
EV	S	N	McL	D			
—	—		4 d.	—	3660		
—	—		—	1	3650.6	27393	S_{18}
—	1			4	3639.8	27474	R_{25}
—	1			9	3627.1	27570	Q_{28}
3	2			15	3618.4	27636	P_{31}
2	3			12	3609.4	27705	R_{26}
—	—		1	2	3593.8	27826	
—	—		1	2	3587.1	27878	R_{27}
—	—		1	—	3572		
—	1			8	3562.5	28070	P_{32}
2	3			20	3530.9	28321	S_{25}
—	1			7	3518.8	28419	R_{28}
—	1			7	3514.0	28458	P_{33}
—	—		1	-	3489		
1	1			8	3481.3	28725	P_{35} S_{27}
1	1			8	3476.9	28761	
—	1			7	3468.7	28830	P_{36}
—	—		—	2	3457.8	28920	Q_{32}
10	2			1	3447.5	29006	Arc-line
	3			—	3446.5		»
6	3			12	3440.5	29065	
1	2			10	3433.7	29123	P_{37}
—	1			8	3422.4	29219	
1	—		—	4	3421.5	29227	
2	2			10	3404.7	29371	P_{38}
—	1			7	3393.2	29471	P_{39}
6	4			10	3385.3	29539	Q_{34}
6	4			10	3381.4	29573	Q_{35}
1	3			10	3374.0	29638	P_{40}

TABLE I (*Continued*).

Intensity.					λ	ν	Remarks.
EV	S	N	McL	D			
—	6			15	3364.7	29720	
1	8				3363.4	29732	S_{29}
—	2			7	3359.1	29770	R_{32}
—	2			7	3357.2	29787	S_{30}
8	5		6	12	3345.8	29888	P_{41}
—	—		3	1	3337.7	29961	
1	—		—	—	3326.4		
1	4		3	9	3322.1	30101	P_{42}
3	4		5	9	3312.8	30186	S_{31}
—	—		3	7	3302.0	30284	
3	3		5	9	3291.1	30385	R_{34}
—	4		—	9	3289.1	30404	
—	3		3	7	3278.8	30499	P_{43}
—	2		3	6	3262.0	30656	
—	—		3	1	3258.8	30686	
—	—		—	2	3253.9	30732	Q_{41}
—	2		3	6	3241.1	30854	
1	1		1	5	3224.8	31010	S_{33}
2	2		—	8	3220.9	31047	
—	1		—	1	3219.1	31064	R_{38}
2	1		—	—	3217.5		Arc-line
1	4		—	5	3209.1	31161	R_{39}
—	—		4	—	3205.6		
1	3		—	6	3202.1	31230	S_{34}
2	2		—	6	3190 6	31342	R_{40}
—	2		5	6	3188.3	31365	S_{36}
—	—		2	—	3174.0		
1	1		4	3	3170.0	31546	
1	1		3	1	3157.6	31670	S_{37}

TABLE I (*Continued*)

Intensity.					λ	ν	Remarks.
EV	S	N	McL	D			
—	—		2	—	3148.6		
3	—		—	—	3143.7		
4	2		4	5	3129.5	31954	
5	4		6	6	3105.4	32202	R_{43}
1	1		—	—	3103.1		
1	1		2	—	3074.6		
1	—		—	—	3067.3		
6	5		5	5	3062.6	32652	S_{42}

said arc-lines might possibly have to be attributed to a less good observational power in the extreme red and violet, appears therefore unfounded.

The importance of the ground spectra made it desirable to perform new measurements. The best method to obtain the first spark spectrum of potassium in great purity and completeness we found to be exciting the luminosity of very diluted incandescent potassium vapour under the influence of very rapidly varying electrical forces.

When our investigation was in progress, there appeared a publication by Mc Lennan [1]) on the spectrum of ionized potassium. His tables present a close resemblance to ours, but in his Table I Mc Lennan gives the lines which he has observed besides those of Schillinger. Hence he also finds the arc-lines, which we succeeded in eliminating.

Besides both in his and in Schillinger's observations a few important lines are wanting. Important because they have been serviceable in the search for the regularities to be mentioned presently. By the aid of Table I it is possible to compare the measurements of our second (D) with those of the other observers, besides the data in the column "remarks" show which P, Q, etc. could only be determined by the new lines. At the same time it is at once clear which of the lines are arc-lines. We estimate the accuracy of the measurements

[1]) Mc Lennan, Proc. R. S. Vol. 100. 182. 1921.

from 4700 Å at 0,2 Å. To some lines a ± is added to show that they are not sufficiently accurate. (Cf. Table I).

Argon can emit two types of spectra. One is the so-called red spectrum, which is formed under the influence of weak electric forces, and must, therefore, be called the arc-spectrum of argon. The other is formed by strong electric discharges, and is called the blue spectrum because of its colour; it is the spark spectrum of argon. No spectrum series are known in the red spectrum, but it exhibits the regularity found by RYDBERG [1]) that for $\lambda < 4704$ Å the frequencies of almost all the lines may be arranged in a Table the four columns of which present a constant difference. PAULSON [2]) extended these results to the less refrangible part of the spectrum. RYDBERG's and PAULSON's tables are reproduced here in Table II, somewhat abbreviated, but with continuous notation. It gives the constant differences for the wave-lengths of λ 9233—3319 Å. (Cf. Table II). The relations are:

$$B = A + 846{,}1$$
$$C = A + 1649{,}3$$
$$D = A + 2256{,}1$$

The frequencies in Table II followed by an M have been taken from MEGGERS [3]). They are more accurate than the frequencies in the original tables of RYDBERG and PAULSON. For this reason the mean value of MEGGERS has been put at the head of the $\Delta\nu$-column and not the mean value of all $\Delta\nu$'s.

The spark spectrum of potassium possesses the same property RYDBERG found in argon, for the examined region between 6594—3063 Å. This appears from Table III, which has been obtained by the aid of the data in Table I. Under the heading "Remarks" in Table I the lines inserted and arranged in Table III are indicated by symbols (See Table III).

The relations for the lines of ionized potassium are:

$$Q = P + 847$$
$$R = P + 1695$$
$$S = P + 2542$$

The first spark spectrum of potassium is, therefore, still somewhat simpler than the red spectrum of argon, the differences being:

[1]) RYDBERG. On the constitution of the red spectrum of argon. Astroph. Journ. Vol. **6**. 338. 1897.

[2]) PAULSON, Physik. Z. S. **15**. 831. 1914.

[3]) MEGGERS, Scientific Papers, Bureau of Standards N°. **414**, 1918

TABLE II. Arc-Spectrum of Argon. (RYDBERG and PAULSON).

N°.	A	$\Delta\nu$ 846.2	B A + 846.2	$\Delta\nu$ 803.1	C A + 1649.3	$\Delta\nu$ 606.8	D A + 2256.1
1					10353.2	607.3	10960.5
2			11533.6 M	803.1	12336.7 M	606.8	12943.5 M
3	10837.7 M			(1649.3)	12487.0 M	606.8	13093.8 M
4			11896.7			(1410.4)	13307.1
5	11731.9 M	846.2	12578.1 M			(1409.9)	13988.0 M
6	11889.9 M			(1649.2)	13539.1 M	606.8	14145.9 M
7	12096.6 M	846.2	12942.8 M	803.0	13745.8 M	606.9	14352.7 M
8	12477.0					(2258.1)	14735.1
9	13326.2					(2258.5)	15584.7
10					15012.9	606.7	15619.6
11	13668.4	847.9	14516.3			(1410.3)	15926.6
12					15429.3	606.7	16036.0
13	14223.7			(1651.8)	15875.5		
14			15078.3			(1409 9)	16488.2
15	14413.4			(1651.2)	16064.6		
16			15398.6			(1409.6)	16808.2
17					16219.8	606.9	16826.7
18					16340.6	606.5	16947.1
19			15699.2	803.5	16502.7		
20	14972.3			(1651.7)	16624.0		
21					16716.2	607.3	17323.5
22			16029.3	802.7	16832.0		
23			16130.5			(1409.7)	17540.2
24			16144.0	803.1	16947.1		
25			16164.2			(1409.9)	17574.1
26			16431.4	802.7	17234.1		
27			16481.3			(1409.7)	17891.0
28			16520.9	802.6	17323.5		
29	15699.2	847.6	16546.8				
30	15787.2	847.3	16634.5				
31	15853.1	848.1	16701.2				

TABLE II (*Continued*).

No.	A.	△ν 846.2	B A + 846.2	△ν 803.1	C A + 1649.3	△ν 606.8	D A + 2256.1
32			16866.1	803.4	17669.5		
33	16298.2	847.7	17145.9				
34	16334.7			(1651.1)	17985.8		
35	16617.8			(1651.2)	18269.0		
36			17863.9			(1409 6)	19273.5
37			18373.8			(1410.4)	19784.2
38			18474.7	802.5	19277.2		
39	18098.7			(1651.7)	19750.4		
40	21260.2 M	846.1	22106 3 M	803.1	22909.4 M	607.0	23516.4 M
41	21599.5 M			(1649.3)	23248.8 M	606.8	23855.6 M
42	21751.9 M	846.2	22598.1 M	802.7	23400.8 M	606.8	24007.6 M
43	21783.8 M			(1649.2)	23433.0 M	606.9	24039.9 M
44	22163.2 M			(1649.2)	23812.4 M		
45	23013.3	846.4	23859.7	803.3	24663.0		
46	23059.9 M	846.1	23906.0 M	803.1	24709.1 M	606.8	25315.9 M
47	23069.2 M	846.1	23915.3 M	803.2	24718.5 M	606.9	25325.4 M
48	23477.0			(1649.8)	25126.8		
49	24794.8	845.8	25640.6	(1650.0)	26444.8	607.2	27052.0
50					26486.7	606.2	27092.9
51	25675.3	846.8	26522.1	(1649.9)	27325.2		
52					27448.2	606.8	28055.0
53	25864.2	846.6	26710.8	(1649.3)	27513.5	606.9	28120.4
54					27527.2	606.7	28133.9
55	26077.2			(1649.2)	27726.4		
56	27208.3	846.7	28055.0			(1410.1)	29465.1
57			28063.4			(1410.0)	29473.4
58	27242.1	846.6	28088.7	(1649.7)	28891.8	607.1	29498.9
59	27779.2	845.9	28625.1	(1649.7)	29428.9		
60					29518.6	606.8	30125.4
61	27992.3			(1649.8)	29642.1		
62	28201.2	846.3	29047.5				

TABLE III. First Spark-Spectrum of Potassium.

N°.	P	$\overline{\Delta\nu}$ 847	Q = P + 847	$\overline{\Delta\nu}$ 848	R = P + 1695	$\overline{\Delta\nu}$ 847	S = P + 2542
1	± 15165	± 844	16009			±(2550)	± 17715
2	16340			±(1724)	± 18064		
3	18286			(1692)	19978		
4	19778			(1682)	21460		
5	20230	849	21079				
6	21758	848	22606	849	23455	846	24301
7	22195	847	23042	846	23888	849	24737
8	22384	845	23229			(1695)	24924
9	22444					(2549)	24993
10			23337	848	24185	846	25031
11			23665			(1694)	25359
12	22909	850	23759	840	24599	864	25463
13	23204					2539	25743
14	24102			(1708)	25810		
15	24429	852	25281				
16					26310	847	27157
17	24737			(1696)	26433		
18	24845			(1701)	26546	847	27393
19	24882	844	25726				
20	24993			(1708)	26701		
21	25050	844	25894	850	26744		
22	25171			(1695)	26866		
23	25210			(1694)	26904		
24	25485			(1707)	27192		
25	25779	845	26624	850	27474	847	28321
26	26009			(1696)	27705		
27	26187			(1691)	27878	847	28725
28			27570	849	28419		
29	27192					(2540)	29732
30	27246					(2541)	29787
31	27636					(2550)	30186

TABLE III (*Continued*).

N°.	P	$\overline{\Delta\nu}$ 847	Q = P + 847	$\overline{\Delta\nu}$ 848	R = P + 1695	$\overline{\Delta\nu}$ 847	S = P + 2542
32	28070	850	28920	850	29770		
33	28458					(2552)	31010
34			29539	846	30385	845	31230
35	28725	848	29573				
36	28830					(2535)	31365
37	29123					(2547)	31670
38	29371			(1693)	31064		
39	29471			(1690)	31161		
40	29638			(1704)	31342		
41	29888	844	30732				
42	30101					(2551)	32652
43	30499			(1703)	32202		

$1 \backsim 847$, 2×848, 3×847. From this ensues that Table III is not unequivocally determined, like II, because when e.g. only P and Q occur in a row, they can now equally well be placed in another row in the Q and R or R and S columns.

It makes the impression that the number $847 = D$ has a physical meaning, as also a value 846,2 occurs in the argon spectrum, which may possibly be a more accurate value for D.

One more detail of the experiments deserves to be mentioned. In some cases the argon spectrum was observed in the potassium tube at the same time with the first spark spectrum of potassium. We have not to do here with a case of transmutation of potassium into argon, but with the penetration of atmospheric air, of which the argon has been finally left. When, however, all precautions are taken, the spark spectrum of potassium is emitted without argon lines.

Mathematics. — "*Explanation of some Interference-Curves of Uni-axial and Bi-axial Crystals by Superposition of Elliptic Pencils.*" (Third paper.) By J. W. N. Le Heux. (Communicated by Prof. Hendrik de Vries.)

(Communicated at the meeting of March 25, 1922).

Some well-known interference-curves, f.i. the hyperbola's and the lemniscates are obtained by superposition of two equal unissons, under certain conditions, as was remarked in my first paper[1]).

From this observation we may derive a parameter-equation for both cases, which enables us to construct the curves in a simple manner.

The axes being at right angles, the unisson may be given by

$$x = r \cos 2\varphi$$
$$y = r \cos 2(\varphi + \alpha).$$

Each value of the phase-difference 2α corresponds to an ellipse; when we suppose, that this phase-difference increases each time with $2\alpha = \frac{\pi}{2n}$, the unisson has n ellipses.

With regard to an easy construction, the angle 2φ may also be supposed to increase with $\frac{\pi}{2n}$.

The two equal unissons, partially covering each other, are given by:

$$\left.\begin{aligned} x &= r \cos 2\varphi + a \\ y &= r \cos 2(\varphi + \alpha) + a \end{aligned}\right\} \quad \ldots\ldots \quad (I)$$

$$\left.\begin{aligned} x &= r \cos 2\varphi' - a \\ y &= r \cos 2(\varphi' + \alpha') - a \end{aligned}\right\} \quad \ldots\ldots \quad (II)$$

where a is constant and $< r$.

The distance between the centres is $2a\sqrt{2}$.

By altering 2α (and also $2\alpha'$) from 0 to $\frac{\pi}{2}$, the image of the hyperbola's is obtained, and from $\frac{\pi}{2}$ to π, that of the lemniscates.

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[1]) These Proceedings Vol. XXIII, p. 1223—1225.

Each curve of the moiré-image corresponds to a certain constant difference (or sum) of phase.

The equation will first be derived for a constant difference of phase $2\alpha - 2\alpha' = 2\theta$.

This condition, together with (I) and (II) gives:

$$\left.\begin{aligned} \frac{x-a}{r} &= \cos 2\varphi \\ \frac{x+a}{r} &= \cos 2\varphi' \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (III)$$

$$\left.\begin{aligned} \frac{y-a}{r} &= \cos 2\varphi \cos 2\alpha - \sin 2\varphi \sin 2\alpha \\ \frac{y+a}{r} &= \cos 2(\varphi'-\theta)\cos 2\alpha - \sin 2(\varphi'-\theta)\sin 2\alpha \end{aligned}\right\} \quad \ldots \quad (IV)$$

Eliminating 2α from (IV) by means of the relation

$$\sin^2 2\alpha + \cos^2 2\alpha = 1$$

we get:

$$\begin{vmatrix} \frac{y-a}{r} & -\sin 2\varphi \\ \frac{y+a}{r} & -\sin 2(\varphi'-\theta) \end{vmatrix}^2 + \begin{vmatrix} \cos 2\varphi & \frac{y-a}{r} \\ \cos 2(\varphi'-\theta) & \frac{y+a}{r} \end{vmatrix}^2 = \begin{vmatrix} \cos 2\varphi & -\sin 2\varphi \\ \cos 2(\varphi'-\theta) & -\sin 2(\varphi'-\theta) \end{vmatrix}^2$$

or after reduction:

$$\cos^2\{2(\varphi-\varphi') + 2\theta\} - 2\frac{y^2-a^2}{r^2}\cos\{2(\varphi-\varphi') + 2\theta\} + 2\frac{y^2+a^2}{r^2} - 1 = 0 \quad (V)$$

When in this equation $\cos\varphi$ and $\cos\varphi'$ are replaced, resp. by $\frac{x-a}{r}$ and $\frac{x+a}{r}$, we get the equation of the moiré-image in xy coordinates.

It is, however, preferable to seek parameter equations.

Suppose $2(\varphi-\varphi') + 2\theta = 2\Delta$, then (V) becomes:

$$r^2\cos^2 2\Delta - 2(y^2-a^2)\cos 2\Delta + 2(y^2+a^2) - r^2 = 0 \quad . \quad (VI)$$

which gives for y:

$$y = \pm\sqrt{r^2\cos^2\Delta - a^2\cot g^2\Delta} \quad \ldots\ldots \quad (VII)$$

The value of x follows from:

$$2(\varphi-\varphi') + 2\theta = 2\Delta$$

$$\cos 2\varphi \cos 2\varphi' + \sin 2\varphi \sin 2\varphi' = \cos 2(\Delta-\theta)$$

or, with regard to (III) and after reduction:

$$r^2\cos^2 2(\Delta-\theta) - 2(x^2-a^2)\cos 2(\Delta-\theta) + 2(x^2+a^2) - r^2 = 0.$$

This equation, being of the same form as (VI), we get for x:

$$x = \pm\sqrt{r^2\cos^2(\Delta-\theta) - a^2\cot g^2(\Delta-\theta)}.$$

When the original angles φ, φ', α and α' are again introduced, the parameter equations become:

$$\left.\begin{aligned} x &= \pm \sqrt{r^2 \cos^2(\varphi-\varphi') - a^2 \operatorname{cotg}^2(\varphi-\varphi')} \\ y &= \pm \sqrt{r^2 \cos^2\{(\varphi-\varphi')+(\alpha-\alpha')\} - a^2 \operatorname{cotg}^2\{(\varphi-\varphi')+(\alpha-\alpha')\}} \end{aligned}\right\} \quad (VIII)$$

For a constant sum of phase, we find the same equations by changing φ' and α' into $-\varphi'$ and $-\alpha'$.

In both cases, the image is the reflexion of the part in the first quadrant with regard to the axes.

Characteristic is the function

$$f(\varphi) = \sqrt{r^2 \cos^2\varphi - a^2 \operatorname{cotg}^2\varphi}.$$

which is real for $\sin\varphi > \frac{a}{r}$.

It has an initial value 0 for $\varphi = bg\sin\frac{a}{r}$, a fast reached maximum for $\sin^2\varphi = \frac{a}{r}$ and it becomes for this maximum $= r-a$.

This is in accordance to the fact, that the circumscribed squares of the partially covering unissons have a common square with sides $= 2(r-a)$, in which square the moiré-image is inscribed.

For the more general case:

$$x = r_1 \cos 2\varphi + b \qquad x = r_1 \cos 2\varphi' - b$$
$$y = r_2 \cos 2(\varphi+\alpha) + a \qquad y = r_2 \cos 2(\varphi'+\alpha') - a$$

we find:

$$x = \pm \sqrt{r_1^2 \cos^2(\varphi-\varphi') - b^2 \operatorname{cotg}^2(\varphi-\varphi')}$$
$$y = \pm \sqrt{r_2^2 \cos^2\{(\varphi-\varphi')+(\alpha-\alpha')\} - a^2 \operatorname{cotg}^2\{(\varphi-\varphi')+(\alpha-\alpha')\}}.$$

Construction of the Hyperbola's.

The construction is similiar to that, used for a Lissajous-curve, that is: straight lines are drawn parallel to the scaled axes of an orthogonal system and the points of intersection are joined diagonally.

Fig. 1 shows a diagram of the funtion

$$f(\varphi) = \sqrt{r^2 \cos^2\varphi - a^2 \operatorname{cotg}^2\varphi}$$

for $a = 8$, $r = 30$. φ is given in units of $\frac{90}{24} = 3\frac{3}{4}°$ and so, the unisson has $\frac{24}{2} = 12$ ellipses.

The maximum ordinate is $r-a = 22$, for $\varphi \pm 30°$

$$(f(30°) = \sqrt{483} \text{ while } 22^2 = 484).$$

The initial value of $\varphi = 15°$, $\frac{a}{r}$ being $= \pm \frac{4}{15}$.

Between the initial and the maximum value of φ, there are but three ordinates and so, the scaled axes have three dividing-points and the image has three interference-curves, each consisting of four equal parts (Fig. 2).

In the formula, $\varphi - \varphi'$ increases from 15° to 30° and the phase-difference $\alpha - \alpha'$ from $-15°$ to $+15°$.

The constructed curves may be compared to the experimental curves in fig. 3, obtained by superposition of two equal unissons,

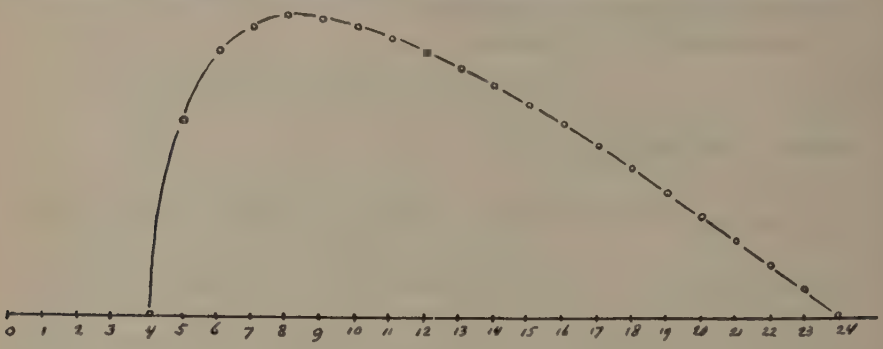

Fig. 1.

each containing 12 ellipses. A much finer result is obtained with unissons of f.i. 50 ellipses, or by comparing to constructed unissons in superposition — these drawings, however, require much time.

It will be evident, that an image with more interference-curves may be obtained by interpolating a *same* number of curves between two succeeding curves of fig. 2.

Construction of the Lemniscates.

This construction is more difficult than that of the hyperbola's, because the image, going to the centre, shows three different species of curves, viz.: ovals, flattened ovals and hyperbola's with doubled ovals.

Only the outer curves are seen in the case of few isophasic lines; they are as easily to construct as the hyperbola's, viz.: by joining the points of intersection, but now following the other diagonal (fig. 4), according to a phase-difference, that begins with 90°.

The constructed curves of fig. 4 may be compared to the experimental curves of fig. 5, the unissons having 12 ellipses each. Fig. 5 is somewhat irregular, owing to the small number of ellipses.

A new difficulty arises from observing, that the axes of co-ordinates are not axes of symmetry for the image of the lemniscates, as is required in the found formula. Still, this image was built in

some experiments, while another time, under apparently the same conditions, a family of ovals appeared. At last, it was found, that the angle between the planes of the pendulums caused the difference: the image of the lemniscates is not built, unless this angle differs from 90° and with a very large number of ellipses per unisson. So in fig. 6, where the angle between the directions of the two composing movements is ± 145° and each unisson has ± 120 ellipses.

The reason for this large number of ellipses proceeds from the swift rising of the function in fig. 1. Dividing-points near the centre are not obtained, unless the interval 4—5 is divided into f. i. 15 parts, corresponding to a phase-difference of $\frac{1}{4}° = 15'$ and a number of 180 ellipses per unisson.

The experimental number however is limited in consequence of the thickness of the ink-lines.

The phaenomenon is mathematically explained as follows:

The unisson

$$x'' = r \cos 2\varphi.$$
$$y'' = r \cos 2(\varphi + \alpha)$$

upon a system with angle 2β, becomes upon an orthogonal system with the same bisectrix:

$$\left.\begin{aligned} x' &= r \sin(\beta + 45°) \cos 2\varphi + r \cos(\beta + 45°) \cos 2(\varphi + \alpha) \\ y' &= r \cos(\beta + 45°) \cos 2\varphi + r \sin(\beta + 45°) \cos 2(\varphi + \alpha) \end{aligned}\right\} \quad (IX)$$

When $\beta + 45° = \gamma$ and seeking the equations of the moiré-image in a similiar manner as before, the composing (oblique) unissons are:

$$x = r \sin\gamma \cos 2\varphi + r \cos\gamma \cos 2(\varphi + \alpha) + a$$
$$y = r \cos\gamma \cos 2\varphi + r \sin\gamma \cos 2(\varphi + \alpha) + a$$

and

$$x = r \sin\gamma \cos 2\varphi' + r \cos\gamma \cos 2(\varphi' + \alpha') - a$$
$$y = r \cos\gamma \cos 2\varphi' + r \sin\gamma \cos 2(\varphi' + \alpha') - a$$

and a point of the moiré-image has the parameter-equations:

$$\frac{x \sin\gamma - y \cos\gamma}{\cos\gamma - \sin\gamma} = \pm \sqrt{r^2 (1 + \sin 2\gamma) \cos^2(\varphi - \varphi') - a^2 \operatorname{cotg}^2(\varphi - \varphi')}$$

$$\frac{x \cos\gamma - y \sin\gamma}{\cos\gamma - \sin\gamma} =$$

$$= \pm \sqrt{r^2 (1 + \sin 2\gamma) \cos^2\{(\varphi - \varphi') + (\alpha - \alpha')\} - a^2 \operatorname{cotg}^2\{(\varphi - \varphi') + (\alpha - \alpha')\}}$$

Now

$$x = \pm \sqrt{r_1^2 \cos^2(\varphi - \varphi') - a^2 \operatorname{cotg}^2(\varphi - \varphi')}$$
$$y = \pm \sqrt{r_1^2 \cos^2\{(\varphi - \varphi') + (\alpha - \alpha')\} - a^2 \operatorname{cotg}^2\{(\varphi - \varphi') + (\alpha - \alpha')\}}$$

where

$$r_1 = r\sqrt{2} \,.\, sin\,(\gamma + 45°)$$

is a moiré-image of two orthogonal unissons.

The constant factor $cos\,\gamma - sin\,\gamma$ only alters the magnitude. When

$$x\,sin\,\gamma - y\,cos\,\gamma = \bar{\bar{x}}$$
$$x\,cos\,\gamma - y\,sin\,\gamma = \bar{\bar{y}}$$

it follows, that

$$x = \frac{\bar{y}\,cos\,\gamma - \bar{\bar{x}}\,sin\,\gamma}{cos\,2\,\gamma} = \frac{sin\,\gamma}{cos\,2\,\gamma}\,.\,(\bar{\bar{y}}\;cotg\;\gamma - \bar{\bar{x}})$$

$$y = \frac{\bar{\bar{y}}\,sin\,\gamma - \bar{\bar{x}}\,cos\,\gamma}{cos\,2\,\gamma} = \frac{sin\,\gamma}{cos\,2\,\gamma}\,.\,(\bar{\bar{y}} - \bar{\bar{x}}\;cotg\;\gamma)$$

Omitting the constant factor $\frac{sin\,\gamma}{cos 2\gamma}$, that does not alter the form, we find at last, that the moiré-image for oblique unissons proceeds from that for orthogonal unissons by the linear substitution

$$x_1 = -\bar{\bar{x}} + \bar{\bar{y}}\;cotg\;\gamma$$
$$y_1 = \quad \bar{\bar{y}} - \bar{\bar{x}}\;cotg\;\gamma.$$

The form, thus chosen, gives rise to an easy construction, executed in fig. 7. The new ordinate, f. i. is found by drawing from a point $P\,(\bar{\bar{x}}, \bar{\bar{y}})$ a straight line, that builds an angle $= \gamma$ with the ordinate of P.

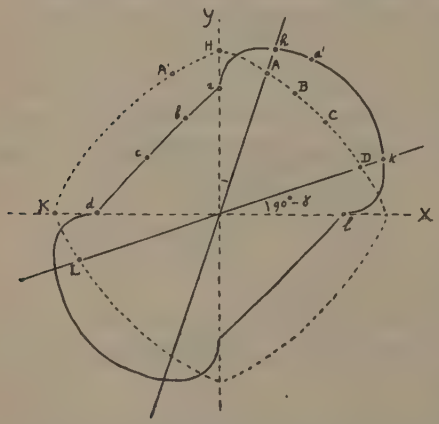

Fig. 7.

By this construction, the double symmetry is lost; the axes turn to each other over an angle $90° - \gamma$.

In fig. 7, a flattened oval is obtained [1]); when the original curve lies nearer to the centre and turns its convex side to the axes, the hyperbola's [2]) are built.

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[1]) See the experimental curves in my first paper, fig. 4.

[2]) A mathematical explanation of interference-curves, wholly different from the here given, is to be found in Mr. T. K. CHINMAYANANDAM: On Haidinger's Rings in Mica. Proc. Royal Society. Vol. XCV, p. 176—189.

The author maintains the pure hyperbola's and the ovals of Cassini, which, however, build a rather rough approximation.

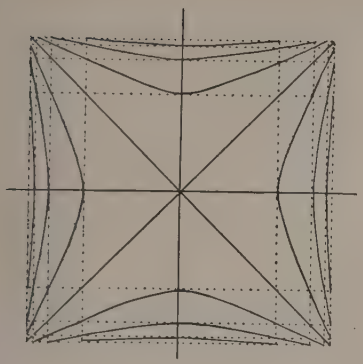

Fig. 2.

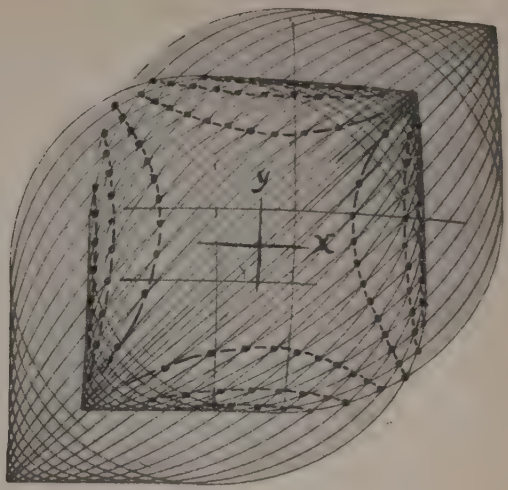

Fig. 3.

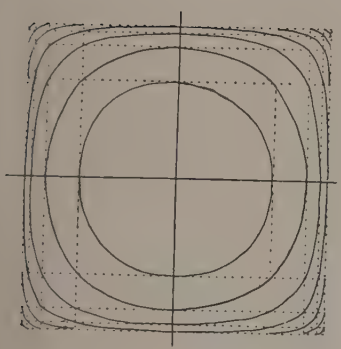

Fig. 4.

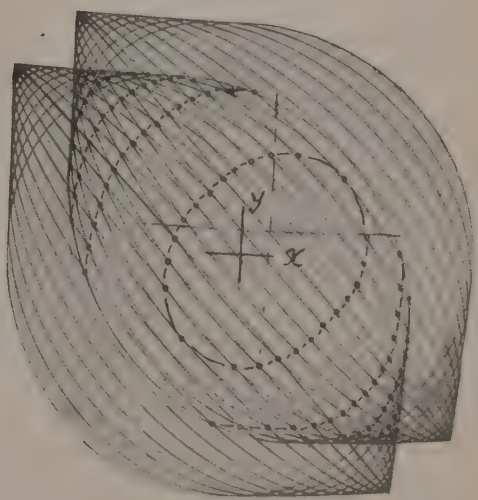

Fig. 5.

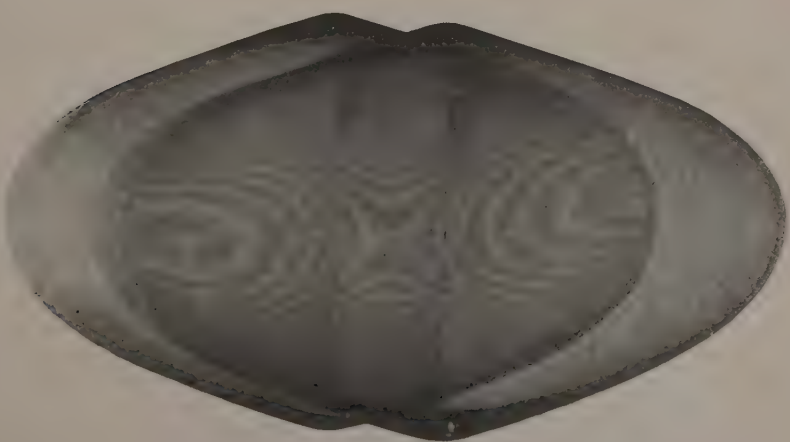

Fig. 6.

Bacteriology. — "*Studies on the bacteriophagus of* D'HERELLE." II. By J. W. JANZEN and L. K. WOLFF. (Communicated by Prof. C. EYKMAN).

(Communicated at the meeting of March 25, 1922).

II. *The Bacteriophagus with regard to flagellates.*

We have been informed by D'HERELLE that the water of some Indian rivers possess the bacteriophagus properties.

In connection with that we have considered it of importance to see how far flagellates out of a mixture of bacteria and bacteriophagus also eat the latter.

In order to do this we prepared a suspension of dead typhoid bacilli in saltsolution, and to a third part of this we added 2 cM^3 canalwater; a second portion was mixed with bacteriophagus and 2 cM^3 canalwater; a third portion was only mixed with the same quantity bacteriophagus as the second.

After 9 days the two first portions had become considerably clearer and we could distinctly show flagellates in the microscopic preparation.

Now dilutions were made, the number of bacteriophagus germs of which was stated in the wellknown way.

We found:

II emulsion + canalwater + bacteriophagus in 1/400.000 cM^3	71 islands.
III emulsion + bacteriophagus in 1/400.000 cM^3	380 islands.

With another trial we found after 14 days:

II emulsion + canalwater + bacteriophagus in 1/4000 mill. cM^3	120 islands.
III emulsion + bacteriophagus in 1/400 mill. cM^3	50 islands.

This numbers are of the same range; the differences range within the mistakes of the experiments.

The suspensions without canalwater remained absolutely turbid, because the bacteriophagus does not affect dead bacilli.

From these two experiments we wish to conclude that the bacteriophagus is not being affected by flagellates.

III. *Constancy of the bacteriophagus properties.*

In our first communication we have proved that various bacteriophagus strains behave differently with regard to different typhoid bacilli.

Here follows a comparition of the bacteriophagus Sm in the sixth and tenth generation with regard to four different typhoid strains; the bacteriophagus was always fed with typhoid Sm.

1. Clearing. 2. Checking. 3. Islandformation.

	6th generation			10th generation		
Strains	1	2	3	1	2	3
Wi	—	++	—	—	++	—
23	++	++	+++	++	++	+++
24	—	—	—	—	—	—
25	++++	++++	++++	++++	++++	++++

So here we see an absolute conformity.

The behaviour of bacteriophagus Sm with regard to strain Wi is somewhat strange; in some generations we did not find any effect; in some others as above mentioned we did find shecking of the growth in broth, but no islandformation.

We have now observed whether the properties of the bacteriophagus change when it is cultivated on different bacteriastrains.

In the following tabels the results are given in which

I. Bacteriophagus Re direct from faeces,
II. ,, Re after having been fed with typhoidbacilli Sm,
III. ,, Wi direct from faeces,
IV. ,, Wi after having been fed with typhoidbacilli Wi,
V. ,, Wi after having been fed with typhoidbacilli Sm,
VI. ,, Sm after having been fed with typhoidbacilli Sm.

The thus obtained bacteriophagi were examined with regard to 5 typhoidstrains.

From this we see that the properties of the bacteriophagus do change when another bacillus has served as food in this sense, that no bacilli which used to affect are now left uninfluenced, but that an increase can appear in the number of strains which are influenced by the bacteriophagus; except for this strengthening however the bacteriophagus retains its specific properties, which in our opinion pleads more for a living being than for a ferment.

1. Clearing. 2. Checking. 3. Islandformation.

I Bact. Re direct from faeces.				II Bact. Re after having been fed with typhoidbacilli Sm.		
Typhoid strains	1	2	3	1	2	3
Wi	—	—	—	—	—	—
1	—	+++	+++	—	++++	++++
24	—	—	—	—	±	++++
27	—	—	—	—	—	—
29	—	—	—	++	+++	++++

III Bact. Wi. direct from faeces.				IV Bact. Wi after having been fed with typhoidbacilli Wi.		
Typhoid strains	1	2	3	1	2	3
Wi	++++	++++	++++	++++	++++	++++
1	—	—	—	—	—	—
24	—	++	++++	—	++	++++
27	—	±	++++	—	±	++++
29	—	++++	++++	—	++++	++++

V Bact. Wi after having been fed with typhoidbacilli Sm.				VI Bact. Sm after having been fed with typhoidbacilli Sm.		
Typhoid strains	1	2	3	1	2	3
Wi	++++	++++	++++	—	—	—
1	++++	++++	++++	—	+++	++++
24	—	+++	++++	—	—	—
27	—	±	++++	—	—	—
29	++	+++	++++	—	—	—

Lab. for hyg. of the University.

Amsterdam, March 1922.

Physics. — G. HERTZ: *"On the Mean Free Path of Slow Electrons in Neon and Argon."* (Communicated by Prof. P. EHRENFEST).

(Communicated at the meeting of March 25, 1922).

The reason for undertaking these measurements was given by researches concerning the efficiency of non-elastic impacts of electrons in neon and argon at potentials just above the excitation-potential. It is known, that those collisions between electrons and the atoms of rare gases, which take place below the excitation-potential characteristic for each gas follow the laws of elastic collisions. As soon as the kinetic energy of an electron surpasses the value corresponding to the excitation potential, it can, on collision with an atom, transfer energy to the latter and thereby raise it from its normal state to a higher quantum-state. This, however, does not take place at every collision between a sufficiently fast electron and an atom; only a certain part, in the case of rare gases most probably only a small fraction, of these collisions is non-elastic and causes excitation of the colliding atom. This fraction we call the efficiency of the particular non-elastic impact. It is equal to the probability that an impact of an electron possessing the required energy really leads to a transfer of energy. It is naturally a function of the velocity of the electron. The form of this function however is not yet known.

In a glow-discharge the two rare gases neon and argon show a characteristically different behaviour, which among other things manifests itself under similar circumstances by producing in neon a much more intensive emission of light than in argon. The reason for this different behaviour according to G. HOLST and E. OOSTERHUIS [1]) probably lies in the fact, that in argon electrons having a velocity above the excitation-potential readily transfer their kinetic energy to the argon-atoms thereby exciting the emission of ultraviolet rays (resonance), while in neon only a small fraction of the impacts leads

[1]) G. HOLST and G. OOSTERHUIS, Physica. 1, 78, 1921.

to radiation the majority of the electrons only imparting their energy to the neon-atoms after falling through a potential-difference equal to the ionization-potential, thus causing ionization.

In consequence one would expect a great difference in the efficiency of the first non-elastic impact in neon and argon. Preliminary experiments concerning the relative value of the efficiency in these gases however have shown, that this difference is not large enough to explain the different behaviour. So there must be another reason. Beside the excitation-potential and the efficiency there is only one quantity which determines the number of the non-elastic impacts, and that is the mean free path of the electrons. Up to now it was assumed, that the value derived from the kinetic theory of gases for particles of infinitesimal small dimensions and large velocity, viz. $4\sqrt{2}$ times the mean free path of a gas-molecule, should hold for the electrons. Recently however, H. F. MAYER [1]) and C. RAMSAUER [2]) have found, from the measurement of the mean free path of electrons, that also for slow moving electrons this quantity depends on the velocity of the electrons, this dependence being different for different gases. Especially between neon and argon RAMSAUER found a very marked difference. While in neon the mean free path depends only to a slight degree on the velocity of the electrons and is nearly equal to the value of the kinetic theory, argon shows for very slow moving electrons, below 1 volt anomalously large values of the mean free path. The mean free path then decreases and becomes a minimum at approx. 12 volts, the minimum being about one third of the value of the kinetic theory. This fact must be of importance for the phenomena produced by electrons passing through a gas, especially in the case of argon, where the mean free path has its minimum value at a potential nearly equal to the excitation potential.

Considering the great importance of the dependence of the mean free path on the velocity, not only for the understanding of the action of electrons in gases, but also for the theory of the atom, it appeared desirable to me, to verify this dependence by direct experiments, in order to obtain accurate values for the ratio of the mean free paths in neon and argon, this ratio being of importance for the evaluation of comparative measurements in the two gases. The applied method is based on the following idea: If in an apparatus of given geometrical dimensions electrons of a certain velocity are allowed to move in a rare gas in a space, in which there is

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[1]) H. F. MAYER, Ann. d. Phys. **64**, 451, 1921.

[2]) C. RAMSAUER, Physik. Zeitschr. **22**, 613, 1921.

no electric field, the mean free path alone will determine their movement and distribution, so long as the velocity of the electrons is not larger than that corresponding to the excitation potential, that is: so long as the impacts are entirely elastic. If the apparatus is then filled successively with different rare gases, the movement of the electrons in the one gas must be the same as that in the other, provided the pressures are chosen in such a way that the mean free path is the same. If, on the contrary, the pressures of both gases has been adjusted so as to make the movement of the electrons the same, the inverse ratio of the corresponding pressures will give the required ratio of the mean free paths under equal pressure. This ratio must be found to be independent of the pressure used in the experiments.

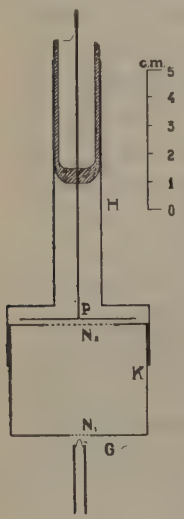

Fig. 1.

The apparatus used is shown in fig. 1. G is a tungsten filament, N_1 and N_2 are grids P is a receiving plate, and H is a metal shield which prevents electrons from coming from G to P by any other way, than through the space between the two grids. All metal parts were made of copper. Before mounting the apparatus they were treated with nitric acid and showed a clean metallic surface after the tube had been exhausted during 5 hours at 400°. The gases used were so pure that no non-elastic impacts, below the excitation potential could be detected even by a very sensitive device.

Before the final measurements, preliminary measurements were made with a simpler device, which differed from that of fig. 1 by omission of the grid N_2. Though the experiments made in this way do not allow an accurate quantitative evaluation, the results are given here briefly, as they show very simply and clearly the different behaviour of neon and argon. During these preliminary measurements the entire apparatus was at earth-potential, except the filament which was brought at a variable negative potential, so as to produce an accelerating electric field between filament and grid. The electron-current passing on to the receiving plate P was measured by a galvanometer. The measurement consisted simply in noting the current as a function of the accelerating potential between G and N_1, in neon and argon under various pressures. In order to be independent of slow variations of the current in the filament, a second galvanometer registered the total electron current, and the quotient of the plate-current and the total

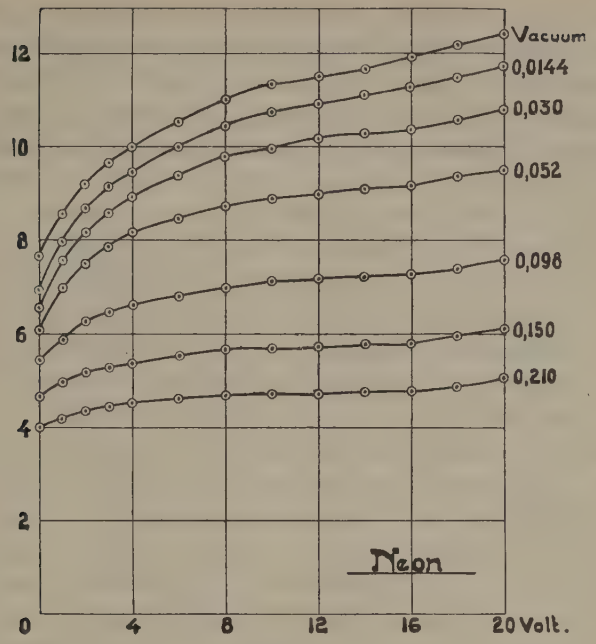
Vacuum
0,0144
0,030
0,052
0,098
0,150
0,210
Neon
12
10
8
6
4
2
0
4
8
12
16
20 Volt.

Fig. 2.

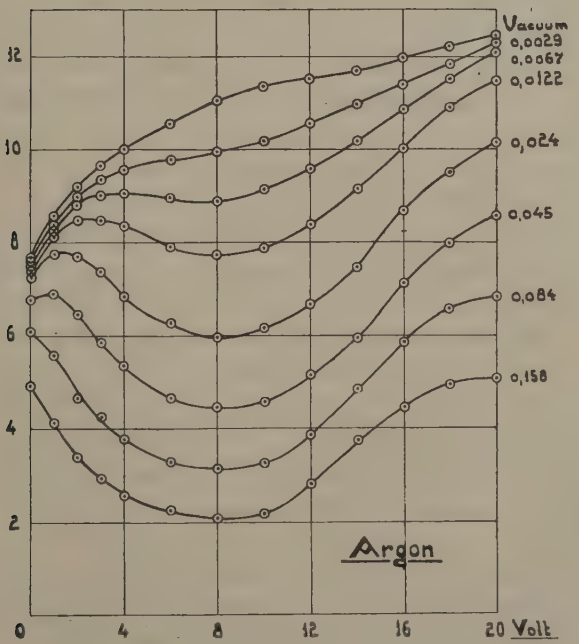
Vacuum
0,0029
0,0067
0,0122
0,024
0,045
0,084
0,158
Argon
12
10
8
6
4
2
0
4
8
12
16
20 Volt

Fig. 3.

electron current from the filament was calculated. As the temperature of the filament was always low, this quotient was independent of the intensity of the electron emission of the filament.

This quotient, multiplied by a constant is plotted in the curves of the figs 2 and 3 for a series of pressures in neon and argon as a function of the potential difference between G and N_1. The numbers near the curves show the gas pressure in m.m. mercury. We see immediately the extra-ordinary difference in the behaviour of both gases. While in neon an increase of pressure for all velocities reduces the plate-current in about the same degree, argon shows at 10 volts a remarkable decrease of current at pressures, where at 1 volt practically no influence is observed. As the observed decrease of current can result only from the collisions between the electrons and the atoms of the gas, we can deduce from these measurements qualitatively, that the mean free path of electrons in argon varies strongly with the velocity of the electrons, while in neon this is not the case, or at any rate only to a small degree. A quantitative calculation in the sense of the above consideration can only be taken from these measurings for slow electrons up to about 10 volts; at higher velocities the electrons produce secondary electron emission from the metalwalls. To retain these secondary electrons, the second grid N_2 was introduced a retarding potential equal to $^3/_4$ of the accelerating potential between G and N_1 being applied between N_2 and P. The result of such series of measurements is shown in figs. 4 and 5 wherein the numbers near the curves again show

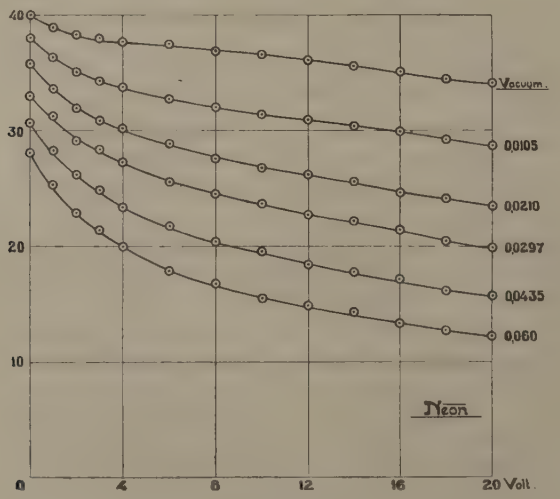

Fig. 4.

the gaspressure in millimetres mercury. For the evaluation of these measurements the distribution of the electron velocities was first measured in vacuo by means of a variable retarding field with the result, that, in consequence of the initial velocity of the electrons, the potential gradient at the filament and the Volta-potential difference

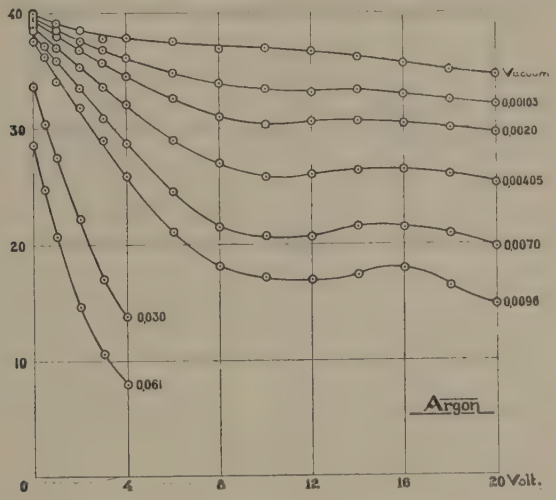

Fig. 5.

between filament and grid, 0.7 volt had to be added to the applied accelerating potential, in order to obtain the true velocity of the electrons. For a series of electron velocities the logarithm of the plate-current was registered as a function of the pressure in neon and argon. A similar character of the curves in neon and argon is to be expected, assuming that the method is correct, in such a way that for each velocity the proportion of corresponding pressures in neon and argon (i.e. pressures giving equal plate-currents) is constant. This is in fact the case for all electron-velocities up to 16 volts. To show this, the curves so obtained for a number of velocities are reproduced in fig. 6. The evaluation is simplified by the fact that the first part of the curves is straight. From the slope of these straight portions we can obtain directly the ratio of the corresponding pressures and so also the ratio of the mean free paths of the electrons.

A condition for the correctness of the method here applied is, that all collisions between electrons and atoms are absolutely elastic. By reason of the very low efficiency of the non-elastic impacts below the ionization potential in the rare gases this is no doubt the case for

potentials between the excitation- and the ionization-potential and for the low pressures used here. Things are different above approx.16 volts, the ionization potential of argon. This already can be observed at the curves for argon at higher pressures in fig. 5, by a bend in the curves at 16 volts; consequently the ratio of corresponding pressures is no more accurately constant there, as is to be seen in fig. 6 at the curves for 18 volt. At the same time this curve shows,

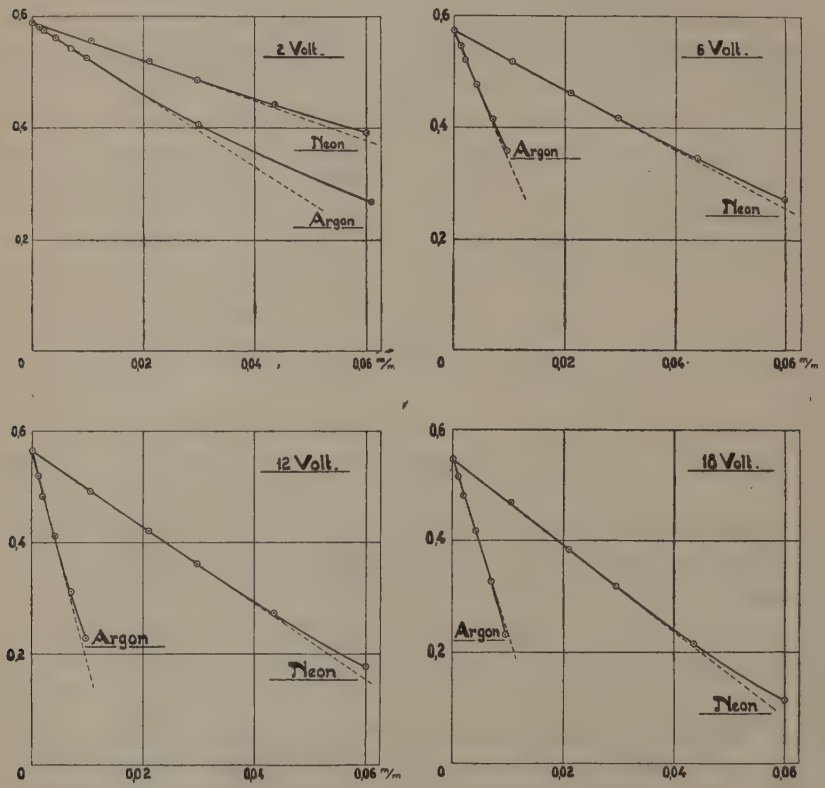

Fig. 6.

that for the lower pressures the number of ionising impacts is so small as to play no part, so that there is no objection against deducing the ratio of the mean free paths from the ratio of the slopes of the first straight parts of the curves.

As a result of the measurement, the values for the ratio of the mean free paths of electrons in neon and argon obtained in this way are shown in fig. 7 as a function of the potential corresponding to the velocity of the electrons; in fig. 8 they are plotted as a function of the root of this potential, being proportional with the

velocity of the electrons. The dotted line in fig. 8 shows for comparison the values of this ratio as deduced from RAMSAUER's measurements. It will be seen that our measurements verify not only the fact of the variation of the mean free path of the electrons with their velocity, as found by RAMSAUER, but also the general character of this variation. The maximum of the curves was found in the present measurements at a potential about 2 volts less than in RAMSAUERS.

The action of the slowest electrons is theoretically of special interest. As however the accuracy of such measurings decreases for extremely slow electrons an extrapolation in the direction of the

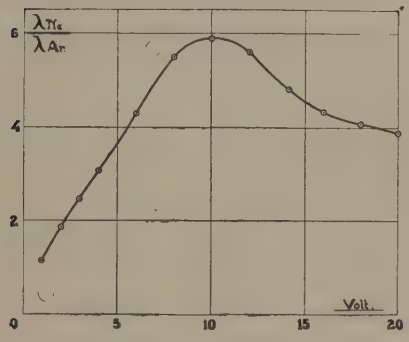

Fig. 7.

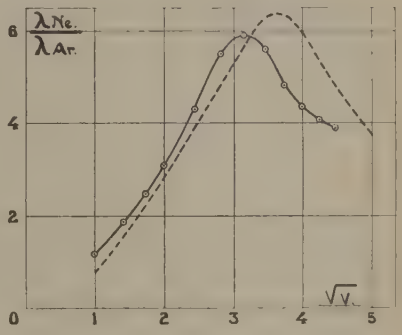

Fig. 8.

velocity zero is always doubtful. If we stipulate, according to the results of RAMSAUER that the electrons in neon show nearly normal values of the mean free path, it appears that, according to the here obtained results, the mean free path of electrons in argon on approaching zero-velocity, do not reach an infinite value, but one about 3 times that calculated from the kinetic theory for very rapidly moving particles of infinitesimal small dimensions. This figure can however, by no means lay claim to accuracy.

The number of collisions of an electron passing through a unit length under the influence of an electric field E, in a gas, in which its mean free path is λ, is $\frac{1}{E\frac{e}{m}}\frac{v^2}{\lambda^2}$, that is, inversely proportional of the square of the mean free path. In argon the mean free path, just below 12 volts, the excitation potential, reaches its minimum of about $^1/_3$ of the value derived from the kinetic theory. We can therefore conclude that an electron of this velocity in argon in passing

through a length unit makes about 9 times as many collisions as would be expected from the kinetic theory, while in neon the number of collisions is nearly normal. This shows clearly, why in argon non-elastic impacts above the excitation potential have a marked effect, while under similar conditions in neon they are hardly noticeable.

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Anatomy. — *"On the morphology of the testis of Rana fusca Rösel"* By G. J. van Oordt. (Communicated by Prof. J. Boeke.)

(Communicated at the meeting of April 29, 1922.)

Introduction.

In recent years several investigations have given us a better insight into the course and the structure of the seminiferous tubules of a number of Mammals and of one Bird (cock). Formerly it was tried to isolate these tubules by the process of maceration and teasing in order to establish their form, their mutual relation and their connection with the rete testis. The results were not convincing, however, because it could not be traced with certainty whether the free ends found were natural or had originated by tearing.

By means of complete series of sections and wax-reconstructions Bremer (1911) succeeded in disclosing the complicated structure of the embryonic human testis. He discovered that the testis tubules form a closed network. Employing a new, good injection method, followed by maceration and teasing, Huber and Curtis (1913) isolated in the testis of the adult rabbit several arch-shaped seminiferous tubules, connected to the rete testis with both extremities. Besides these simple "single-arched" (n-like) tubules, "double-arched" (m-like) tubules, connected with the three free ends to the rete, were met with. Relatively simple tubules as well as canal-systems of complicated structure were found in the rabbit's testis; canals terminating in blind ends or diverticula were not described, however. Applying the same method Huber (1916) discovered in the testis of the cock that the seminiferous tubules form a network, in which no blind ends occur.

Studying complete series of sections Curtis (1913) met with various single-arched tubules in the testis of the mouse. Anastomoses between two arches occur but rarily. Later on (1918) Curtis investigated the testes of mouse, rabbit and dog and in these animals he also found the simple n-like tubule to be the original one. However, the testis of the mouse shows the simplest structure, then the testis of the dog and next that of the rabbit follows as to complication.

Independent of Curtis, de Burlet and de Ruiter (1920) came to the same results in studying a number of complete series of sections

of testes of mouse-embryos of 9—17 mm. length. The fundamental form of the embryonic testis tubule is a simple n-like tube, of which the convex side is directed towards the periphery and of which the extremities are connected with the future rete testis. A number of these tubes are placed serially behind each other; anastomoses between the arches and double-arched, m-like tubes occur also. The plane of the arch is perpendicular to the longitudinal axis of the testis. Tubules, terminating in blind ends, were rarely found. In the caudal part of the testes of embryos of 13 mm. and smaller a so-called „canal-complex" occurs, from which later on — for in older testes more arches are to be found than in younger ones — additional arches probably develop. The tubules number from 10 to 13 in the mouse. After the „canal-complex" has disappeared, the longitudinal growth of the testis-tubules sets in and then the tubules begin to coil strongly. From the longitudinal stem, originally epithelial, the rete testis develops.

In a second paper DE BURLET (1921) traced the morphology of a few Marsupialian testes (*Perameles obesula*, *Didelphys* spec., *Halmaturus Bennetti*). The single-arched tubule was found again; in *Perameles* the testis (embryo of 50 mm.) is still more simply built than in the mouse; the testis of *Didelphys* (embryo of 20 mm.) is composed of two long, strongly twisted tubules. These tubules are very numerous in *Halmaturus* (embryo of 105 mm.), where they vary from 200 to 300.

Starting from the above investigations it was but natural to trace in one of the representatives of the other Vertebrate groups, how the shape of the adult seminiferous tubule derives from the embryonic one. After consulting Dr. H. M. DE BURLET, to whom I wish to express my thanks for his interest in this work, I chose the common Frog, *Rana fusca* Rösel. As it appeared during the investigation that in immature frogs the course of the vasa efferentia, the ducts through which later on the spermatozoa pass to the kidney, show different peculiarities, I decided to communicate simultaneously a few remarks concerning the course of these channels in immature frogs in the beginning of their second year.

Material and methods.

All specimens of the common frog were caught at Bilthoven (near Utrecht) in Sept. 1920. The smallest, immature frogs measured 2.8 cm. (from the head to the rump), the largest, adult spec. 6.3 cm. According to GAUPP (1904, III, pp. 298—300) frogs measuring circ.

30 mm. are in their second, those measuring circ. 50 mm. in their third year, whilst they become mature in the end of the fourth year.

The gonads of the immature frogs were taken from the body, together with the kidney; they were fixed in BOUIN's solution and after 5 days they were transferred to alc. 90 %. Subsequently the testes were cut — mostly frontally, but in a few cases transversely — into complete series of sections of 10 μ. The sections were generally stained with DELAFIELD's hematoxylin and VAN GIESON's solution, sometimes eosin or nigrosin was used instead of VAN GIESON's solution. Especially with VAN GIESON's solution the connective tissue between the seminiferous tubules assumes a deep red colour.

From the testes of the adult frogs only the middle part was sectioned, from all other testes complete series of sections were made. As far as necessary, the sections were drawn on transparent paper at a magnification of 100, with the aid of the large projection-apparatus of ZEISS.[1]) By laying these transparent papers on each other, it is generally not difficult to trace the course of the tubules, which are cut transversely. Originally I had the intention to project on a certain plane several tubules, passing over into the rete testis with a common stem, but in many cases this method proved not practicable, especially in adult testes, as here the tubules are too close to each other and too much twisted. Fig. 10 is even so schematized that only the mutual relations of the tubules, drawn in one plane, are shown. In order to get an exact insight into the course of the seminiferous tubules a few sections of the part of the testis, in which these tubules occur, are also reproduced.

In the following the development and structure of the testis tubules are described in the first place and further the particular course of the vasa efferentia in six immature testes is treated.

The development and structure of the seminiferous tubules.

An extensive literature deals with the development of the gonads of frog-embryos. As most of these investigations do not bear upon my subject, I will only communicate the results of WITSCHI, who in his „Experimentelle Untersuchungen über die Entwicklungsgeschichte der Keimdrüsen von *Rana temporaria*" (1914) not only traced the different developmental stages of the gonad, but also drew attention

[1]) I have to thank Prof. A. J. P. VAN DEN BROEK, whose kindness enabled me to use the apparatus of the Anatomical Institution of the University at Utrecht.

to the morphology of the testis tubules of newly metamorphosed frogs.

After describing the development of the so-called indifferent gonad — which possesses a germinal epithelium consisting of one layer and surrounding a central lumen, the primary genital space, in which cell-strands, the sexual strands, situated at regular distances behind each other, have originated from the mesonephros — WITSCHI traces the development of the ovary and the direct testis-development. The indirect testis-development, which takes place in the so-called hermaphrodites of PFLÜGER is elaborately described; in this case the testis originates from an ovarium-like gonad[1]). As this development does not bear directly upon my subject and as the final stage of both direct and indirect testis-development is the same, I will not enter any further upon this question. Shortly, the direct testis-development is as follows. The germ cells leave the germinal epithelium, wander through the primary genital space and settle on the sexual strands. All germ cells leave about simultaneously, so that only the peritoneum remains. Between the germ cells and the compact core of the sexual strands several slits originate: the anlages of the lumina of the testis-ampullae. Then the ampullae differentiate from each other and in this way the anlages of the testis tubules develop. These ampullae are short, almost globular tubules, with a lumen disappearing later on.

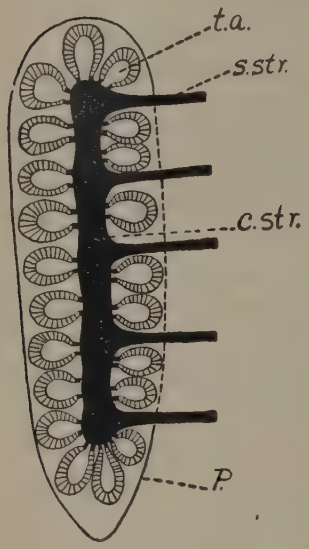

Fig. 1.
Schematized longitudinal section of the testis of a newly metamorphosed frog. After WITSCHI (1914).

The convex side of the ampullae is directed towards the periphery of the testis; with the other side they are attached to the central strand. The sexual strands are connected with the mesonephros. The distal ends of these strands thicken, fuse and in this way the central strand originates in the longitudinal axis of the juvenile testis. After some time the inner-testicular network or rete testis originates from the central strand, as well as the vasa efferentia from the compact sexual strands.

A schematized longitudinal section of the testis of a newly meta-

[1]) WITSCHI's latest publication (1921) treats the same subject.

morphosed frog is reproduced in fig. 1, which is drawn after WITSCHI (1914, fig. *A*, p. 21.)

In the literature, dealing with the further development of the testis, only some scattered remarks on the testis tubules are to be found. "Damit (i.e. when the stage, reproduced in fig. 1 is reached) haben die Samenkanälchen im wesentlichen ihren definitiven Zustand erreicht" (WITSCHI 1914, p. 20). Then the testis ampullae grow out "zu den bekannten schlauchförmigen und gewundenen Samenkanälchen, während sich die Keimzellen ziemlich rasch vermehren" (WITSCHI 1914, p. 20). However, nothing is mentioned about this outgrowth and about the question whether the tubules are connected with each other.

GAUPP describes the form of the testis tubules of the adult frog as follows (1904, III, p. 307): "Sie beginnen an der Oberfläche gerade und mit radiärer Anordnung gegen das Centrum hin, laufen dagegen mehr central vielfach gewunden durch einander. Die radiären Canalabschnitte der peripheren Zone beginnen blind unter der Tunica albuginea, und häufig sieht man hier, wie zwei gesondert entstehende sich mehr central mit einander vereinigen".

It is my intention to trace how the structure of the adult testis originates from the simple one of newly metamorphosed frogs, the latter having been described by WITSCHI.

I started with the study of testes of frogs in the beginning of the second year. It proved easiest to get an insight into the form of the testis tubules by studying frontal testis-sections, in which a great number of transversely cut tubules are visible (cf. figs. 2, 3, 6, 7, 8, 9). These sections were drawn on transparent paper and then compared.

In figg. 2 and 3 parts of two frontal sections of the right testis (long 1.8, broad 1 mm.) of a common frog with a head-rump length of 3.5 c.m. are reproduced. Fig. 2 is a section close to the rete; many tubules transversely cut, are distinctly visible. On tracing the course of the three tubules, designated *A*, *B* and *C*, to the periphery, we observe that in most cases these tubules branch, like the fingers of a hand, into a number of tubules (fig. 3, which is drawn after a section close to the periphery) and that all these tubules are

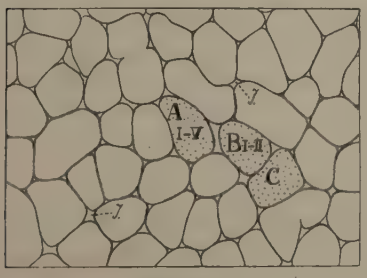

Fig. 2.

Section of the testis of a juvenile frog (beginning of second year), near the rete testis (× 100).

terminating in blind ends. Tubule *A* ramifies into five tubules (*A I*, *A II*...*A V*), *B* into two tubules (*B I*, *B II*), while *C* remains single. Already in fig. 2 it is visible that the tubules *A* and *B* divide into a certain number of branches, for these tubules are designated *A I—V* and *B I—II* in this figure. On comparing figs. 2 and 3 we see that the space between the tubules, the interstitium, is larger near to the rete than towards the periphery. As has already been mentioned, fig. 3 is drawn after a section close to the testis-surface, so that not all testis tubules are cut transversely. Many tubules, which were not cross-cut, were indistinctly visible and for this reason this part of the section is shaded by oblique lines.

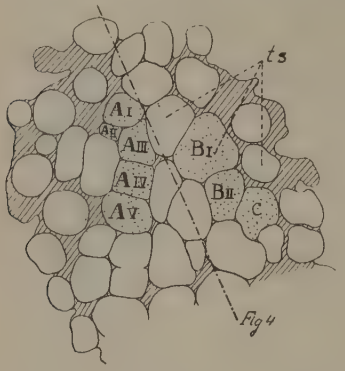

Fig. 3.
Section as in fig. 2, but more near the periphery (× 100).

To elucidate the course of the seminiferous tubules, I have projected the circumferences of the testis tubules *A*, *B* and *C* on a sagittal plane of the testis (this plane is marked by a —.—.— line in fig. 3). This is reproduced in fig. 4, in which the course of these tubules can be seen. Moreover it is visible that the ducts of the rete into which the tubules *A*, *B* and *C* pass over, are directly connected with each other.

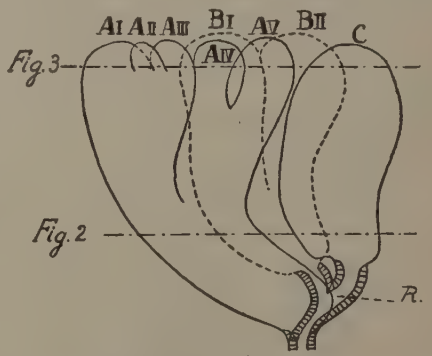

Fig. 4.
Projection of the tubules (designated in figs. 2 and 3) on a sagittal testis-plane (× 150).

The left testis of the same frog was cut transversely. The testis tubules are built in the same manner as those of the right testis. However, on comparing the form of the tubules of the cranial and caudal part of the left testis with the form of the tubules of the middle part of the right testis, we see that in the former the number of simple, not branching tubules is much larger than in the latter. A peculiarity of this left testis is that the most caudal vas efferens is not connected with the rete testis. In this testis there is a small caudal part, consisting only of three testis tubules, which do not open into the rete, but are directly connected with the mesonephros

by an efferent duct. In fig. 5 this is figured. Only these caudal vasa efferentia are projected on the mid-sagittal plane of the testis. Moreover, two single testis tubules, directly passing over into the rete testis, are sketched; the vas efferens, previous to the last, gives off a side branch to the last efferent duct, but a connection is not established, however.

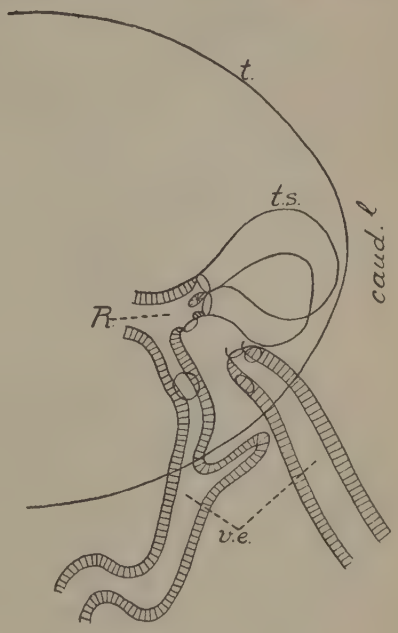

Fig. 5.

Projection of the two posterior vasa efferentia on the sagittal longitudinal testis-plane. Frog from the beginning of the second year (× 100).

Two frontal sections of the left testis (long 6, broad 3,5 m.m.) of a frog in the beginning of the third year (4,75 c.m. in length) are reproduced. Fig. 6 shows a section, close to the rete testis, of which different parts are visible. The tubules *A*, *B*, *C* and *D* are separately connected with the rete; tubule *B* just ends in the rete in the section reproduced; in a neighbouring section tubule *C* is connected with this same rete canal. In this figure arrows indicate with which part of the rete a few of the testis tubules are connected. When we trace the course of the testis tubules, indicated *A*, *B*, *C* and *D* towards the periphery, we see that here also these tubules divide into many others; e.g. tubule *A* splits up into seven, *B* into five, *C* into four and *D* into six others. Fig. 7 shows a section of the same testis about halfway the periphery. At this level tubule *A* has divided into 3 branches (*A I—II*, *A III—V*, *A VI—VII*), *B* into three (*B V*, branched off nearer to the rete is very short), *D* into four, while tubule *C* has not divided as yet; this will take place closer to the periphery. The space between the tubules, being rather wide near the rete, is very narrow at this level. Most tubules end near the periphery; anastomoses are never found.

On comparing the testis of a newly metamorphosed frog (fig. 1) with that of a second or third-year one (figs. 2—7), we find that the testis tubules, which are single originally and terminate in blind ends, divide already in the second year into a number of branches (like the fingers of a hand) and that this subdivision has increased in the

third year. The testis having strongly increased in size during this time, it is impossible that several testis-ampullae have fused to form such a canal system. On the contrary, we must conclude from the stages, described above, that the testis tubules, which are originally simple and very short and which are called testis ampullae then, divide toward the periphery into a number of tubules and that these branches are connected with the rete by the proximal part of the ampulla. On comparing the different sections [1]) we see that both length and diameter of the seminiferous tubules have strongly increased.

Fig. 6.
Section of the testis of a frog of the third year; the rete is partly visible (× 50).

Turning now to the testis of the adult frog, we observe almost the same here. In figs. 8 and 9 parts of two frontal sections of the left testis (long 10.5, broad 7 mm.) of an adult common frog (length 6.3 cm.) are reproduced. The tracing of the course of the strongly ramified testis tubules and the graphic reconstruction of this taking too much time, I can only describe a few tubules, not very strongly branched. They are reproduced in figs. 8 and 9 and a reconstruction of the same tubules, beside each other and in one plane is given in fig. 10. This had to be done, because the tubules, winding too much around each other, especially in the neighbourhood of the rete, could not be reproduced, projected on a certain plane.

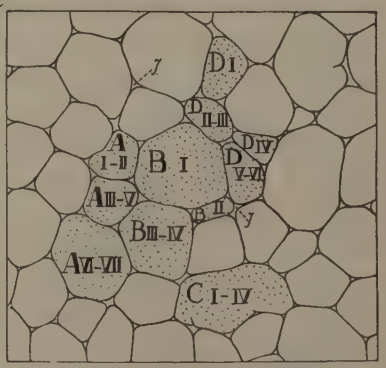

Fig. 7.
Section as in fig. 6, but about halfway the periphery (× 50).

The tubules, designated *A I* and *A II* in fig. 8 do not branch

[1]) Originally I had the intention to reproduce all the figures at the same magnification (× 100); this proved impossible, however, the figures of immature frog-testes then becoming too small and those of adult frog-testes becoming too large.

further towards the periphery; tubule *B I—II* (fig. 8) splits up into two tubules towards the periphery (fig. 9), while *B III* is very

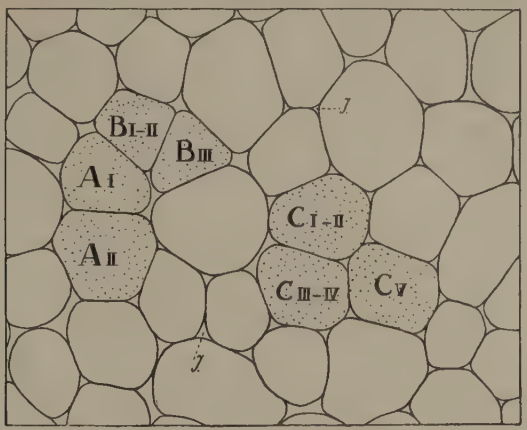

Fig. 8. Section of the testis of an adult frog, about halfway the rete (× 50).

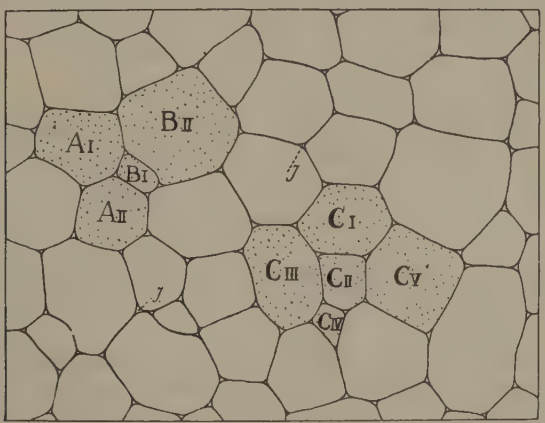

Fig. 9. Section of the testis of an adult frog, near the periphery (× 50).

short and ends blindly about halfway the periphery (fig. 10). If the testis had developed further, this short tubule would probably have grown peripherally. Tubule *C* divides into 5 parts. *A*, *B* and *C* are connected with the rete close to each other.

On comparing this testis with those, described above, we see that apart from the size, there is no fundamental difference in the shape of the tubules. The seminiferous tubules of the adult testis have the same shape, but are longer and thicker. They form no anastomoses and all end blindly. Most of them are strongly branched. The tubules twist, especially near to the rete. Towards the periphery the tubules are situated so close to each other that there is but a

very narrow space left for the interstitium. Towards the rete testis this space increases in width (cf. figs. 9 and 8).

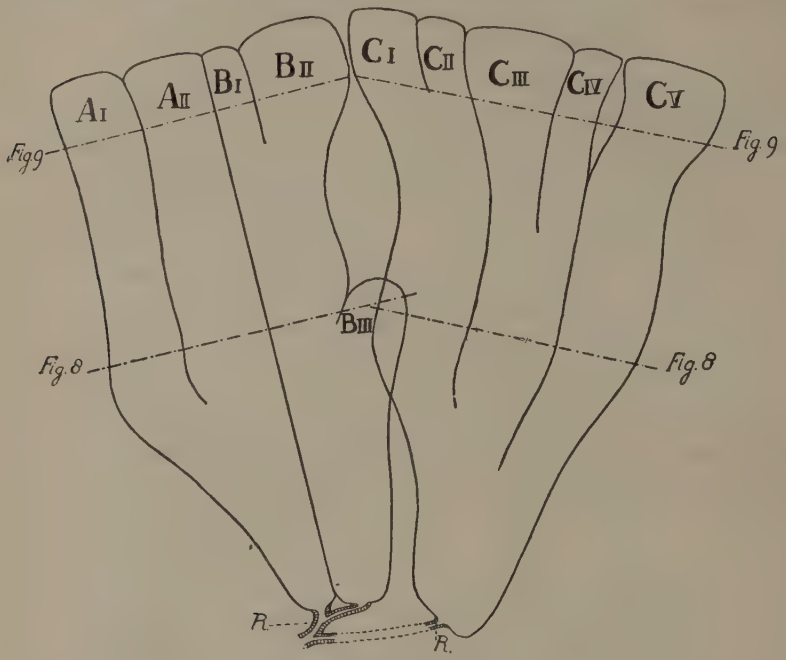

Fig. 10. Schema of the course of the testis-tubules, designated in figs. 8 en 9 (×50).

The course of the vasa efferentia in frogs in the beginning of the second year.

It is generally known that in adult frogs the vasa efferentia, which arise at the medial side of the testis, form a network, the extratesticular network, between testis and kidney. The number of these channels greatly varies. According to Gaupp (1904, III, p. 355) they number from 4 to 11 in *Rana fusca*. These differences are not only individual, but occur also in the right and left testis of one and the same animal. Channels which terminate blindly and do not reach the kidney are numerous, according to Gaupp.

Investigating a number of testes of immature frogs, I found that here these particuliarities were also present. A conspicuous difference is that the extratesticular network has not developed as well as in adult frogs, the vasa efferentia being still situated serially close behind each other in the mesorchium.

I will describe six testes, derived from two frogs of 2.8 cm. and one frog ot 3 cm. in length. With regard to the vasa efferentia,

they show the following particuliarities and differences, sketched schematically in fig. 11 *a—f*; testes, corpora adiposa and kidney are dotted, while rete testis and vasa efferentia are black. For simplicity's sake all the ducts are indicated by successive numbers.

Fig. 11*a* gives a schema of the right testis of a frog, measuring 2.8 cm. in length. From the testis to the mesonephros 4 efferent ducts run, of which the two last have fused over some distance. At the cranial side of the testis there is also a vas efferens (N°. 2), but this one is not connected with the mesonephros. It runs cranialward and ends in the fat body. Still more in front of the fat body there is a very short vas efferens, connected neither with the testis nor with the kidney.

In fig. 11*b* a schema of the left testis of the same juvenile frog, with 10 efferent ducts is reproduced. The most cranial one, running only over a short distance in the fat body, can be compared to the first vas efferens of the right testis of the same frog. The rete testis is connected by 8 different vasa efferentia (N^{os} 2—9) with the mesonephros. N^{os} 5 and 6 arise from the rete at some distance from each other, but quite near to the testis-surface they come close together and run parallel without fusing, however, to the mesonephros. The two ducts (N^{os} 8 and 9) at the caudal side of the testis arise close to each other but separately, from the testis, and unite just outside the testis to form a common duct. As is the case in the testis described above (p. 8) and sketched in fig. 5, the 10th vas efferens is not connected with the rete testis. Only a few seminiferous tubules open into this duct; so these are directly connected with the kidney.

The two testes sketched in fig. 11*c* and 11*d* belonged to a frog, also measuring 2.8 cm. in length. In both the most cranial efferent ducts have no direct connection with the mesonephros, but run cranialward to the fat-body and from here to the kidney. In the right testis, behind this efferent duct, there are still six others, from which N^{os} 3 and 4 are only separated over a short distance, quite near to the kidney.

In the left testis of the same animal the 2^{d} and 3^{d} vasa efferentia arise separately from the rete; they fuse near the mesonephros to form a common duct. The caudal vasa efferentia, N^{os} 5 and 6, run parallel in the testis and unite there, where they leave the testis; then they split, subsequently they again form one duct and finally they enter the kidney separately.

The right testis of the specimen, the last to be described (3 cm. in length), shows only one peculiarity (fig. 11*e*) i.e. the 3rd and 4th

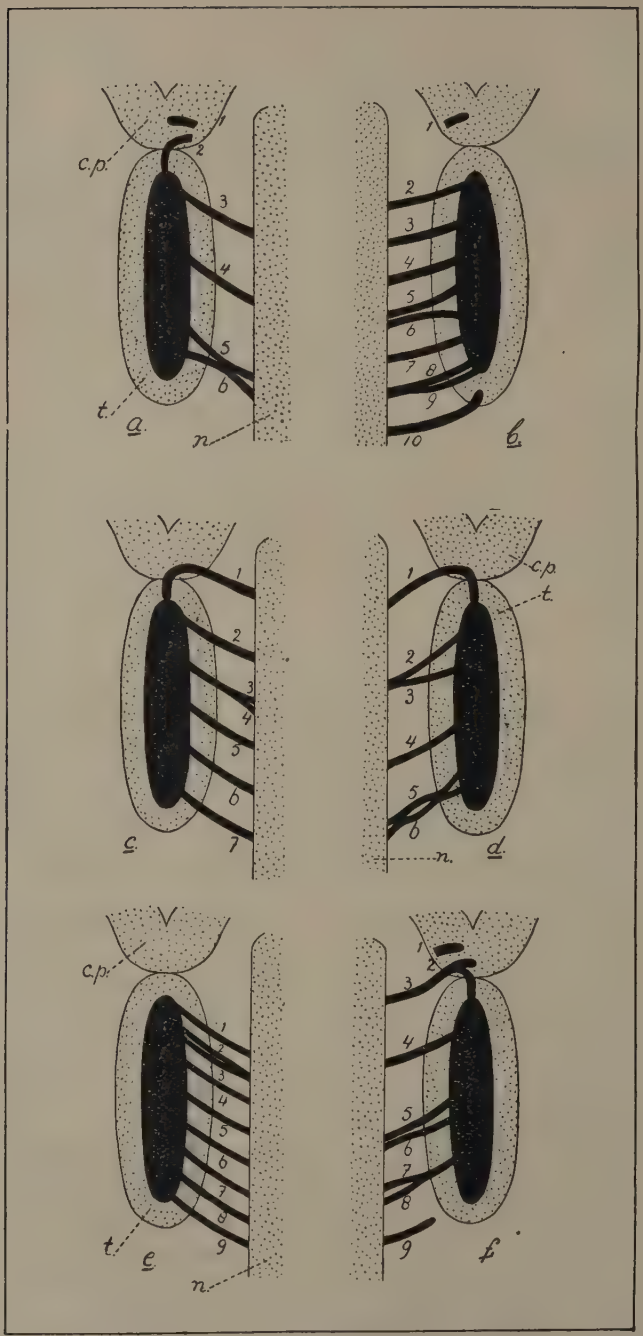

Fig. 11. Schemata of the course of the efferent ducts in 6 testes of juvenile frogs.

vasa efferentia, running close to each other, fuse near the kidney.

The left testis is remarkable for the following facts (fig. 11*f*). The most cranial vas efferens runs like the most cranial ones, sketched in fig. 11*a* and 11*b*; the third vas efferens runs like both cranial efferent ducts of fig. 11*c* and 11*d* and moreover, a short side-duct (N°. 2), coming from the fatbody, opens into it. The vasa efferentia N^{os} 5 and 6 are close to each other, especially outside the testis, but enter the mesonephros separately. The 7th and 8th vasa efferentia leave the rete testis united and split outside it; the 9th vas efferens finally is connected with the kidney, but does not reach the testis.

So we have seen that the course of the efferent ducts in immature frogs is as variable as in adult ones and that there is no symmetry between left and right testis of the same animal.

SUMMARY.

I. According to WITSCHI the testis of a newly metamorphosed *Rana fusca* is composed of a great number of short tubules, the testis-ampullae, which end blindly, and are implanted around and perpendicular to a longitudinal stem, the central strand. With this central strand the mesonephros is connected by the sexual strands. The ampullae, which possess a lumen, disappearing later on, form no anastomoses and are not branched. Later on the rete testis originates from the central strand, the vasa efferentia from the sexual strands.

II. During the further development of the testis, the testis-ampullae increase in length as well as in diameter and they simultaneously divide towards the periphery into a great number of branches, which nearly all grow out till they reach the periphery. Only a few short tubules, not reaching the testis-surface, were noted.

III. The testis-tubules of an adult frog, are composed in the same way: towards the periphery they split up more and more. All tubules terminate in blind ends, and they never form anastomoses. The tubules, which are straight near the periphery, are often somewhat bent and twisted near to the rete.

IV. In two testes of immature frogs it was observed that a small caudal part of the testis is not connected with the rete, but that the tubules, composing it, opened directly into an efferent duct.

V. The courses of the vasa efferentia of six immature frogs in the beginning of the second year show several peculiarities:

1. A real network, as in adult frogs, was not noted.

2. In the fat-body short tubules often occur, neither connected with the testis nor with the kidney.

3. In some cases the cranial part of the rete testis is connected with the kidney by an efferent duct, which first passes through the fatbody. In one case a short side-duct, coming from the fatbody, opened into such a duct.

4. It was often observed that vasa efferentia, which run close together, fuse. This fusion can take place near the testis as well as near the kidney.

5. In a few cases an efferent duct was found, which, originating from the mesonephros, did not reach the testis.

6. The vasa efferentia between rete testis and mesonephros number from four to nine, this agreeing with the number, observed in adult frogs. In the left and the right testis of the same animal the number can vary.

Utrecht, April 1922. *Zoological Laboratory, Veterinary College.*

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ABBREVIATIONS.

c.p. = corpus adiposum, fat body.
c.str. = central strand.
I. = interstitium.
n. = kidney.
P. = peritoneum.
R. = rete testis.
s.str. = sexual strand.
t. = testis.
t.a. = testis-ampulla.
t.s. = tubuli seminiferi, testis tubules.
v.e. = vas efferens.

Mathematics. — *"A New Method for the Solution of the Problem of the Characteristics in the Enumerative Geometry."* By G. SCHAAKE. (Communicated by Prof. HENDRIK DE VRIES.)

(Communicated at the meeting of April 29, 1922).

§ 1. In this paper a general method will be set forth for the determination of the expressions through which the problem of the characteristics in the enumerative geometry is solved. These are the expressions indicating how many individuals two algebraical systems resp. of ∞^p en ∞^{n-p} figures, depending on n parameters, have in common.

The method in question will be best explained by application to a special example. We shall therefore by the aid of it solve the problem of the characteristics for the straight line in a space of an arbitrary number of dimensions. [1])

§ 2. We shall first confine ourselves to the straight lines of a plane V. In V we assume a point C and a straight line c by the aid of which we represent the plane homographically on itself. With a view to this we associate to a point P of V the point P' of the straight line CP that together with C, the point of intersection C' of CP and c, and P' forms an anharmonic ratio that is equal to a constant number λ. Through this transformation a straight line l of V is transformed into a straight line l' cutting l on c.

Especially we consider the transformation for which $\lambda = 0$. In this case for an arbitrary point P the distance $C'P' = 0$, so that the point P' corresponding to a point P generally lies in the intersection of CP with c. If, however, P lies in C, together with the straight line CP also the distance $C'P'$ becomes indefinite, so that to the point C all the points of V are associated.

For an arbitrary straight line l the corresponding line l' coincides with c. If, however, l passes through P, there are ∞^1 associated

[1]) Cf. for other applications Cap. VI of my academical dissertation which will shortly appear, entitled: *Afbeeldingen van figuren op de punten eener lineaire ruimte*, Groningen, P. NOORDHOFF, 1922.

straight lines, which form a plane pencil that has the intersection of l and c for vertex.

If λ changes continuously, out of a system S of ∞^1 straight lines through the homographic representation described above, ∞^1 new systems are derived forming a coherent set, which contains S (for $\lambda = 1$) and of which we shall especially consider S', the system arising from S through the transformation belonging to $\lambda = 0$. The number of straight lines which a system of this set has in common with a system of ∞^1 straight lines not belonging to the set, is apparently independent of λ. In order therefore to know how many straight lines S has in common with another system S^1 of ∞^1 straigh lines, we may equally well investigate the same for S'.

Now any straight line of S is transformed into the line c, which may always be chosen outside S^1. If, however, S contains k straight lines l passing through an arbitrary point, so that k is the class of the curve enveloped by the lines l, the k straight lines of S through C are transformed into as many plane pencils of straight lines l'. S^1 contains k^1 lines of each of these plane pencils, if the straight lines of S^1 envelop a curve of the class k^1. From this we conclude that S' and S^1, hence also S and S^1, have kk^1 straight lines in common.

§ 3. In order to apply the same method to the straight lines of space, we assume a point C and a plane γ, and we make use of the homographic representation arising if to each point P we associate the point P' that forms with C, the point of intersection C' of CP with γ, and P an anharmonic ratio $= \lambda$; in this representation there corresponds to any straight line l another straight line l' cutting l on γ. If again we take the case $\lambda = 0$, to any straight line l a straight line l' of γ is associated, the intersection of the plane (C, l) with γ, unless l passes through C in which case there are ∞^2 associated lines l', which form a sheaf of rays that has the point of intersection of l and γ for vertex.

In this way a ruled surface R is represented in a system R' of ∞^1 rays l' of γ. These envelop a curve of the class ϱ, if ϱ is the order of R. For through a point P of γ there pass those straight lines l' that are the images of the straight lines l of R cutting CP. If now we consider a complex K of the order $\varkappa$, this has in γ ∞^1 rays enveloping a curve of the class $\varkappa$, so that K has $\varkappa\varrho$ rays in common with R'.

A line complex of the order $\varkappa$ has therefore $\varkappa\varrho$ lines in common with a ruled surface of the order ϱ.

Through our transformation a congruence G passes into a system G' that consists first of all the rays of γ, each counted β-fold, if β represents the class of G. For each line l' of γ is associated to the β lines l of G lying in the plane Cl'. Further, if α is the order of G, there are α rays of G which pass through C and are transformed into as many sheaves of rays of G'. Another congruence with the order α' and the class β' has $\alpha\alpha' + \beta\beta'$ rays in common with G'. From this follows the well known theorem of HALPHEN:

Two line congruences (α, β) *and* (α', β') *have* $\alpha\alpha' + \beta\beta'$ *lines in common.*

§ 4. Before we give the general solution of our problem in an R_n, we consider the special case that we have to do with the ∞^6 straight lines of an R_4. By the aid of a point C and a space Γ in R_4, we arrive at the ∞^1 homographic representations that are each characterised by a value of the anharmonic ratio $\lambda = (CC'PP')$ if C' is the point of intersection of CP en Γ. Again we consider especially the representation belonging to $\lambda = 0$.

If we take a system S_1 of ∞^1 rays, this is transformed by the latter representation into a ruled surface S_1' of the order ϱ lying in Γ, if ϱ represents the number of straight lines of S_1 cutting a plane. If we consider further a system S_5 of ∞^5 straight lines of which an arbitrary plane pencil contains $\varkappa$, the rays that S_5 has in common with Γ form a complex of the order $\varkappa$, so that S_5 contains $\varrho\varkappa$ rays of S_1'.

A system S_1 *of the order* ϱ *has* $\varrho\varkappa$ *rays in common with a system* S_5 *of the order* $\varkappa$.

A system S_2 of ∞^2 rays is represented on S_2', a congruence (α, β) of Γ, if α is the number of rays of S_2 cutting an arbitrary straight line (through C), and β the number of straight lines of S_2 lying in an arbitrary space (through C). A system S_4 has a congruence (φ, ψ) in common with Γ, if φ and ψ represent the numbers of straight lines of S_4 resp. belonging to a (three-dimensional) sheaf of rays or lying in a plane. S_4 has $\alpha\varphi + \beta\psi$ rays in common with S_2'.

A system S_2 (α, β) *has* $\alpha\varphi + \beta\psi$ *rays in common with a system* S_4 (φ, ψ).

A system S_3 is transformed through our representation into a system S_3' consisting first of a complex of the order ν lying in Γ, if ν is the number of rays of S_3 lying in a (three-dimensional) special linear complex. Further, if S_3 contains μ rays through a given point, to each of the μ straight lines l through C there are associated the

∞^3 rays l' passing through the point of intersection of l and Γ, so that S_3' contains also μ four-dimensional sheaves of rays. If besides S_3, we have another system S_3^1 with the characteristic numbers μ_1 and ν_1, this has in Γ a ruled surface of the order ν_1 and it contains μ_1 straight lines of each of the four-dimensional sheaves in S_3'. S_3^1 and S_3' have accordingly $\mu\,\mu_1 + \nu\,\nu_1$ rays in common.

Two systems S_3 (μ, ν) *and* S_3^1 (μ_1, ν_1) *have* $\mu\,\mu_1 + \nu\,\nu_1$ *rays in common.*

§ 5. Bij means of complete induction the following results may be easily proved, through which the problem of the characteristics is solved for the straight line in R_n.

The characteristic numbers of a system S_p of ∞^μ rays in R_n indicate how many straight lines of S_p there are in an $R_{n-\mu+1}$ lying in R_n which cut an $R_{n+\mu-p-2}$ in the aforesaid $R_{n-\mu+1}$ for all values of μ satisfying the inequalities: $\mu > 0$, $n + \mu - p - 2 < n - \mu + 1$ or $\mu < \frac{p+3}{2}$ and $n + \mu - p - 2 > -1$ or $\mu > p - n + 1$.

From this follows, that the p-fold number of characteristics for the straight line in R_n, if $p < n$, is equal to $\frac{p+1}{2}$ or $\frac{p}{2} + 1$, according to whether p is odd or even, and for $p \geq n$ equal to $\frac{2(n-1)-p+1}{2}$ or $\frac{2(n-1)-p}{2} + 1$ according to whether p is odd or even. The p-fold number of characteristics is, therefore, equal to the $2(n-1)-p$-fold number.

The expression indicating how many lines an S_p and an $S_{2(n-1)-p}$ have in common, is a polynome of which all the terms are found by multiplying each time those characteristic numbers of S_p and $S_{2(n-1)-p}$ that belong to conditions which together define a straight line in R_n.

§ 6. It is clear that the indicated method may also be applied to the case when we have to do with figures composed of a definite number of points, straight lines, planes etc. If the parts of these figures are independent of each other, it will often be desirable to transform them by different homographic representations.

The system e.g. of the ∞^n groups of n points $(P_1\, P_2, \ldots, P_n)$ of a straight line l may in the following way be represented homographically on itself. We assume on l $2n$ arbitrary points $C_1, \ldots, C_n$, $\Gamma_1, \ldots, \Gamma_n$ and associate to a point P_i of a group of n points (n-group) the point P'_i defined by: $(C_i\, \Gamma_i\, P_i\, P'_i) = \lambda_i$.

If we take all $\lambda_i = 0$, there belongs to an arbitrary n-group the n-group $(\Gamma_1, \ldots, \Gamma_n)$; if, however, a point P_i coincides with C_i, the associated point P'_i, becomes indefinite, so that to an n-group of which the k points $P_{i_1}, P_{i_2}, \ldots, P_{i_k}$ coincide resp. with $C_{i_1}, C_{i_2}, \ldots, C_{i_k}$ there are associated ∞^k groups that have the $n-k$ points $\Gamma_{i_{k+1}}, \ldots, \Gamma_{i_n}$ in common.

Let us now consider a system S_k of ∞^k n-groups with the $\binom{n}{k}$ characteristic numbers $\alpha_{i_1 i_2 \ldots i_k}$, indicating how many groups of the system there are for which the points $P_{i_1}, P_{i_2}, \ldots, P_{i_k}$ are defined. Through our representation S_k is transformed into a system S'_k consisting of $\binom{n}{k}$ separate systems of ∞^k groups. Such a system is formed e.g. by the n-groups that have their points $P_{i_{k+1}}, \ldots, P_{i_n}$ resp. in $\Gamma_{i_{k+1}}, \ldots, \Gamma_{i_n}$ and of which the remaining points P are indefinite. Each group of this system is associated to the $\alpha_{i_1 i_2 \ldots i_k}$ groups of S_k that have their points $P_{i_1}, \ldots, P_{i_k}$ in $C_{i_1}, \ldots, C_{i_k}$ and is therefore an $\alpha_{i_1 i_2 \ldots i_k}$-fold group of S'_k. If we take another system S_{n-k} of ∞^{n-k} groups, with the characteristic numbers $\beta_{i_1 i_2 \ldots i_{n-k}}$, we find from the number of common groups of S'_k and S_{n-k}:

A system $S_k(\alpha_{i_1 \ldots i_k})$ of ∞^k n-groups of points has with a system $S_{n-k}(\beta_{i_1 \ldots i_{n-k}})$ of ∞^{n-k} n-groups $\Sigma \alpha_{i_1 \ldots i_k} \beta_{i_{k+1} \ldots i_n}$ groups in common.

Finally we remark that the expounded method may also be applied to curves, surfaces etc.

Physics. — "*On the diffraction of Röntgen-rays in liquids.*" By Prof. W. H. KEESOM and Prof. J. DE SMEDT. (Communication N°. 10 from the Laboratory of Physics and Physical Chemistry of the Veterinary College). (Communicated by Prof. H. KAMERLINGH ONNES.)

(Communicated at the meeting of March 25, 1922).

§ 1. *Introduction.* The investigation by means of Röntgen-rays of the structure of substances that are in liquid or solid state at temperatures lower than the ordinary one, seems us to be of extraordinary importance. These substances namely belong to those that possess the most simple chemical structure (in the gaseous state several of them are mon- or diatomic). In most cases their molecules consist of light atoms small number of electrons). Therefore the experimental results obtained with these substances will lead more easily than other ones to conclusions of importance for the structure not only of the crystalline state but also of the molecule and the atom.

We thus gladly followed the invitation of Prof. KAMERLINGH ONNES to make such an investigation on the diatomic elements oxygen, nitrogen, if possible on hydrogen etc. and the monatomic elements as f.i. argon. In the discussion of the scheme for this investigation, for which we made at Leiden some preparatory experiments, the first question was the following: Will liquefied gases also give a diffraction figure when they are crossed by a beam of Röntgen rays as it was the case with the liquids that were investigated by DEBIJE and SCHERRER [1])? As some Röntgen-technical difficulties had to be overcome we agreed to continue the preparatory experiments at Utrecht, as far as we should be able to obtain there the liquid gases and work there with them. Some of the results of these experiments will be given in this paper. In these investigations we did not only use liquid oxygen and argon [2]) but also some substances that are liquid at ordinary temperatures.

§ 2. *The apparatus.* Fig. 1 shows the vacuum glass g and fixed

[1]) P. DEBIJE and P. SCHERRER, Nachrichten Göttingen 1916.

[2]) The argon was put at our disposal by N.V. Philips' Gloeilampenfabrieken, for which we wish to express here our thanks.

to it the camera c, into which the liquid gas is poured and in which it will be radiated by the Röntgen beam which is bounded by the diaphragm d of tin (length 34 mm., diameter of the opening 2 mm.) shut by a leaf of aluminium. The lower part of the inner tube is narrow. First it consisted of a small tube of aluminium thick 0,015 mm. and with a diameter of 3 mm. which was soldered to a copper tube by means of wolframine. Later on this aluminium tube was replaced by a glass tube thick 0,002[5] to 0,01 mm. [1]) and with diameter 2 mm., blown to a wider glass tube. Except between b_1 and b_2 the glass was silvered.

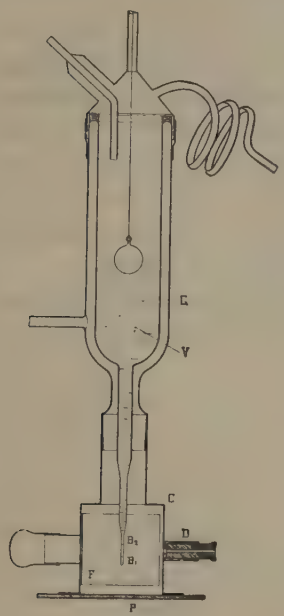

Fig. 1.

The camera (radius 27,5 mm.) is fixed to the outer glass by means of a ground plug. In the camera along the cylinder wall the film f is stretched (Eastman duplitized X-ray film) in the same way as was done by DEBIJE and SCHERRER. For taking in and out the film, which was wrapped up in black paper, the camera was detached from the plate p to which its ground border had been cemented. The vacuum was obtained with a LANGMUIR condensation pump with the rotating mercury pump of GAEDE as a forepump. This vacuum sufficed to expose with one single filling of 200 cM³ of the liquefied gas during more than 5 hours.

The Röntgen-rays were excited by a metal SIEGBAHN tube with Cu-anticathode. The K_β rays were filtered away by a Ni-plate of 0,01 mm. The current given by an inductorium with gas interruptor was ± 10 mA., tension ± 25 KV., time of exposition as a rule 5 hours.

For a photograph of the Röntgen interference figure of ice (see § 3) we used a glass tube partly filled with water. The lower part of this tube consisted again of a thin glass tube as described above. The tube with water was let down into a vacuum vessel with a lower part of thin-walled glass filled with liquid air. During the exposition the tube was rotated from time to time.

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[1]) These thin tubes of aluminium and glass are proofs of the ability of the amanuenses 1st class J. J. VAN DER SLUIS and A. R. B. GERRITSE, the last of whom has also made several of the here mentioned photographs.

The substances that are liquid at ordinary temperatures were exposed in a more simple glass apparatus with a thin walled lower part, which fitted on the same camera, while again the camera was evacuated.

§ 3. *Results.* We have exposed liquid oxygen, liquid argon, benzene, water, aethylalcohol, aethylaether, formic acid, carbonic disulphide, bromium.

Of these carbonic disulphide and bromium (in glass tube) gave no distinct diffraction figure [1]).

The other liquids gave first an intense almost circular diffraction ring. Fig. 2 shows the diffraction ring of oxygen.

Argon was exposed twice, once in an aluminium tube and once in a glass tube. Of these only [2]) the first one gave a distinct diffraction figure.

In table I φ represents the half top-angle of the cone formed by the diffracted Röntgen rays.

TABLE I.

Substance	φ	a	$1.33\sqrt[3]{\frac{M}{d}}$
oxygen	27°	4.0 Å	4.0 Å
argon	27	4.0	4.1
benzene	18	6.0[5]	5.9
water	29	3.7[5]	3.6
aethylalcohol	22	4.9	5.2
aethylaether	19	5.7	6.2
formic acid	24	4.5	4.5

By the agreement between the diffraction rings of oxygen and argon we might come to the hypothesis that these rings are due to the same impurity f.i. to small ice crystals. This was however proved to be not the case. Therefore oxygen namely was first dried by KOH and P_2O_5, then liquefied and destilled in apparatus dried beforehand and finally poured through a filter of cotton wool into

[1]) The probable reason for this is, that the Röntgen rays are absorbed to such a high degree by these substances, that the Röntgen-light diffracted by the liquid on account of its small intensity cannot be distinguished from that diffracted by the glass.

[2]) Probably by the reason mentioned in note 1.

the vacuum glass that was filled with dry air [1]) and in which such a filter was placed again at the entrance of the narrow part. This oxygen now gave the same ring. On the other hand a photograph of ice (see § 2) surrounded by liquid air taught us that none of the interference lines of ice coincide with the ring of oxygen.

The diffraction image of water shows still an interesting detail (see fig. 3 of the plate). Immediately following on the intense diffraction ring the film shows a very considerable almost uniform blackening with a rather sharp outline at $\varphi = 46°$.

For some other liquids too we found weak indications of a similar blackening.

For oxygen and argon the best films show beside the ring given in table I still a weak second ring, for oxygen at $\varphi = 46°$, for argon at $\varphi = 49°$.

§ 4. *The intense diffraction ring is due to the cooperation of neighbouring molecules.* As was shown by EHRENFEST [2]) and at the same time by DEBIJE and SCHERRER (l.c.) a diffraction ring like that of § 3 may be due to the interference of Röntgen-rays diffracted by arbitrarily orientated systems each of two (or more) particles, which have a definite mutual distance (f.i. the two atoms in a diatomic molecule, where each of the atoms is regarded as one single diffracting centre.) Between the angle φ and the distance a of the two diffracting particles we have then (see EHRENFEST l.c.) the following relation

$$a = \frac{7{,}72\,\lambda}{4\,\pi\,\sin\frac{\varphi}{2}}, \quad \ldots\ldots\ldots \quad (1)$$

where λ is the wavelength of the Röntgenrays.

The values of a calculated in this way (with $\lambda = 1{,}54$ Å) are given in table I.

In the first place the fact, that also argon has a similar diffraction ring, involves that, at least for argon, this diffraction ring is not due to the cooperation of atoms in the molecule. [3])

That this is neither the case for oxygen is to be expected by the

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[1]) By a small window v in the vacuumglass we could state that the liquid was perfectly clear.

[2]) P. EHRENFEST. These proceedings Vol. XVII, p. 1184. See also P. DEBIJE Ann. d. Phys. (4) 46, p. 809, 1915

[3]) Unless argon should be more-atomic in the liquid state, which is not made probable by the following.

improbable if not impossible great distance, which the centres of the atoms should have then (see table I).

The distance of the interfering particles calculated with (1) however agrees with the distance of the centres of neighbouring molecules, when we think us these arranged as the centres of spheres packed possibly close together. This distance is found in the last column of tabel I (M = molecular weight, d = density). Small deviations, as far as they do not fall within the limits of experimental accuracy, might be ascribed to deviations from the spherical form or to the circumstance that will be discussed in § 6.

From this we think it justified to draw the conclusion, that the intense diffraction ring found above is caused by the interference of Röntgen light diffracted by neigbouring *molecules* [1] [2]).

For benzene too the above mentioned agreement between a and the distance of neighbouring molecules arranged in closest packing has been stated. From this we think it evident that the above considerations also hold for this substance in contradiction with the opinion of DEBIJE and SCHERRER (l.c.) that this diffraction ring should be due to the atoms in the molecule.

§ 5. When our view that the observed diffraction ring is due to the interference of Röntgen light diffracted by neighbouring molecules is right, the dimensions of these diffracting particles may no longer be neglected compared with their mutual distance and we may ask:

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[1]) This does not involve that we have to do with the cooperation of only two molecules at a time. On the contrary, as far as it is not due to the particular form of the relation between the quantity of Röntgen light and the blackening of the film caused by it, the relative sharpness of the diffraction ring might point at a cooperation of more molecules at a time.

These molecules might then be arranged in the liquid in groups more or less regularly under the influence of the forces which below the melting point condition the regular structure in the crystalline state.

In this way f.i. *both* rings of argon might be explained by assuming that in the liquid a great number of groups is present in which the atoms are arranged in a centered cubical lattice. The mentioned rings correspond then to the planes (110) and (211), the edge of the lattice would be 4,65 Å. For the distance of two neighbouring atom centres follows then again 4,0 Å as in table I.

Because of the perfect analogy in the behaviour of oxygen and argon we should have to replace for oxygen these atom centres by molecule centres. [Later experiments have shown that the ratio of the values of $sin\ ^1/_2\varphi$ for the two rings does not quite agree with the ratio $1 : \sqrt{3}$, as should be the case if the supposition made above were valid. Added in the translation].

[2]) The possibility of this has already been acknowledged by DEBIJE and SCHERRER (l.c.).

in how far may we regard the distance calculated with (1) as the distance of the *centres* of the molecules?

As in reality the electrons are the diffracting particles, this question may only be answered when the true position of the electrons in the molecule is known for every instant.

In order however to form us still an opinion in this problem we shall consider the case of molecules each consisting of a nucleus (which is supposed not to contribute to the diffraction) and one electron that is freely moving in a sphere with radius r (so that it passes in all volume elements equally long times). A system of arbitrarily orientated pairs of such molecules all with the same distance a between the molecule centres gives then in a direction which makes an angle φ with the direction of the incident light an intensity proportional with

$$1 + 9 \frac{\{\sin \alpha r - \alpha r \cos \alpha r\}^2}{\alpha^6 r^6} \cdot \frac{\sin \alpha a}{\alpha a} \quad \ldots \ldots \quad (2)$$

when

$$\alpha = \frac{4\pi}{\lambda} \sin \frac{\varphi}{2} \quad \ldots \ldots \ldots \quad (3)$$

This expression may be easily deduced by an extension of the calculation given by EHRENFEST for the case of two simple diffraction centres.

When r is not small compared with a, the first maximum does no longer correspond with the relation (1). In this case an other factor must be substituted for 7,72 in this formula. When f.i. we take $a = 4$ Å, $r = 1{,}25$ Å, this factor is 7,42.

Evidently the influence of the dimensions of the molecule is small. The more will this be the case as the (mean) density of the electrons in the molecule is greater in the central parts than near the periphery.

When the molecules come so near to each other, that they are in conctact the influence is greater. For the simple molecule models, described above, the factor in (1) would then become 10 % smaller.

§ 6. *Water.* The blackening which is found in the diffraction image for water round the above mentioned diffraction ring seems to point at a rather great number of pairs of molecules with a mutual distance smaller than that wich we shall call here the normal one [1]). On this supposition the limit of this blackening

[1]) With the above is in good agreement, that in table I the mean distance (3,6 Å) for water is smaller than the normal one (3,75 Å).

($\varphi = 46°$) corresponds to the smallest distance between the centres of two neighbouring molecules. Formula (1) gives for this 2,4 Å.

The further examination of the blackening in the diffraction image of the liquids thus gives a direct method of research for the way in which the molecules are distributed in the liquid as to their mutual distances. Some conclusions may be drawn then also on the field of force of the molecules.

The fact, that in water a relatively great number of pairs of molecules occurs with a distance smaller than the normal one will be related with the peculiarities in the thermodynamic properties by which water is regarded as an associating substance. However, we do not find an extraordinarily great number of double or multiple molecules which should have been formed by juxtaposition of simple molecules so as to ly as close as possible to each other.

§ 7. *Oxygen and argon.* By analogous considerations as in § 6 we probably must ascribe the second weak ring for oxygen and argon to pairs of molecules which touch each other [1]).

According to (1) this would give for the distance of the centres for oxygen 2.4 Å, for argon 2.3 Å.

Because of the last remark of § 5 these values might however still undergo a small variation.

Comparing these results with those obtained for water we find: firstly, that in oxygen and argon there is a considerably smaller number of pairs of molecules with a distance below the normal one, secondly that for oxygen and argon in the greater part of these molecule-pairs the molecules are lying together as close as possible.

We might ascribe this different behaviour to a difference in the fields of force: the water should have then a more intense field, which extends over a greater distance, while oxygen and argon should have a field of force which makes itself more felt in the immediate neighbourhood of the molecule. In this way the dipolar character of the water molecule becomes manifest on one hand, the quadrupolar (resp. perhaps octopolar) character of the oxygen and the argon molecule (atom) on the other hand.

[1]) See also p. 122 note 1.

Fig. 2.
Oxygen, with $K\alpha$-rays of copper.

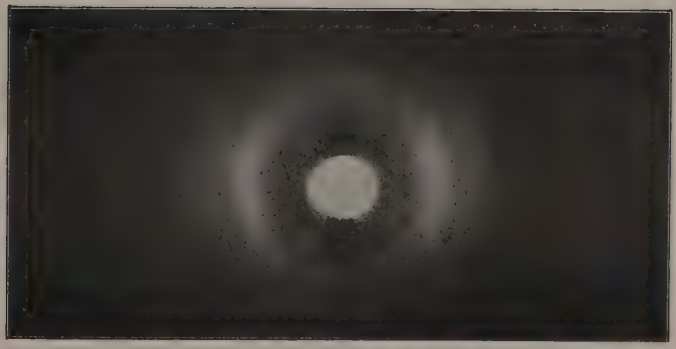

Fig. 3.
Water, with $K\alpha$-rays of copper.

Physics. — "*The crystal structure of germanium*". By Dr. N. H. KOLKMEIJER. (Communication N°. 11 from the Laboratory of Physics and Physical Chemistry of the Veterinary College at Utrecht). (Communicated by Prof. H. KAMERLINGH ONNES).

(Communicated at the meeting of April 29, 1922).

§ 1. *Introduction.* From a medical-biological point of view too, a possibly complete knowledge of the quadruvalent elements as f.i. C and Si will be of great importance. The only one among the elements of the fourth group of the periodic system the crystal structure of which has not yet been investigated is germanium [1]).

For this reason the author undertook the investigation of this structure with the same apparatus that had already been used in the investigation of tin [2]) and in that of $NaClO_3$ and $NaBrO_3$ [3]), that has been described in preceding papers. Only the diaphragm of lead in the camera was replaced by one of tin [4]) while before it a Ni-filter of 0,01 mm. thickness was placed in order to weaken the β radiation from the Cu-anticathode. The germanium (from Dr. TH. SCHÜCHARDT, Görlitz), in the form of a fine powder, was cemented to a thin glass rod, with Canada-balsam.

§ 2. *The crystal structure.* The observations were in good agreement with a structure like that of diamond. In the table this is evident from the satisfactory agreement between the values of $sin^2 \frac{1}{2}\theta$ [5]) derived from the observations with the calculated ones. For the latter we chose as value of the lattice parameter $a = 5{,}61$ Å. From the density at 20°,4 viz. 5,459 [6]), the atomic weight 72,418 [7]) and the number of AVOGADRO $6{,}062 \cdot 10^{23}$ we deduce $a = 5{,}594$ Å.

[1]) As has been remarked by D. COSTER (These Proceedings 21, 1294, 1919) the knowledge of this structure might also be of importance for the question of the eventual existence of binding rings of circulating electrons.

[2]) A. J. BIJL and N. H. KOLKMEIJER. These Comm. Nos. 1 and 2. These Proceedings 21, 405, 494, 1918.

[3]) N. H. KOLKMEIJER, J. M. BIJVOET and A. KARSSEN. These Comm. N°. 5, These Proceedings 23, 644, 1920.

[4]) W. H. KEESOM and J. DE SMEDT. These Comm. Nr. 10. These Proceedings 25, 118, 1922. By a sufficiently high tension the L-radiation of the Pb might namely be excited by the heterogeneous radiation of the Cu, which would cause a blackening of the film. For tin this is much less probable.

[5]) θ is the angle between the rays incident on the substance and those diffracted by it.

[6]) CL. WINKLER, Journ. f. prakt. Chem. 34, 177, 1887.

[7]) J. H MÜLLER, Journ. Am. Chem. Soc. 43, 1085, 1921.

Observed.		Calculated [1].				
		α-lines.			β-lines.	
Intensity	$sin^2 1/2\theta.10^3$	h_1 h_2 h_3	$sin^2 1/2\theta.10^3$	Intensity	h_1 h_2 h_3	$sin^2 1/2\theta.10^3$	Intensity
vs	59	1 1 1	56	1.3			
vf	124				2 2 0	122	1.5
s—vs	153	2 2 0	151	1.5			
fff	168				3 1 1	167	1.1
s—vs	207	3 1 1	207	1.1			
f	249[5]				4 0 0	243	0.4
f	296	4 0 0	301	0.4	3 3 1	289	0.6
m—s	363	3 3 1	358	0.6	4 2 2	363	1.0
s—vs	454	4 2 2	452	1.0			
f (double)	511	3 3 3 / 5 1 1	508	0.6	5 3 1 / 4 4 0	532[5] / 487	0.7 / 0.4
f	599[5]	4 4 0	603	0.4	6 2 0	609	0.6
s—m	652	5 3 1	659	0.7	5 3 3	654	0.3
m	745	6 2 0	753	0.6	5 5 1 / 7 1 1	776	0.5
f	801	5 3 3	810	0.3			
ff	848				6 4 2	852	0.9
f	892	4 4 4	904	0.2	5 5 3 / 7 3 1	898	0.6
s—m	948	5 5 1 / 7 1 1	960	0.5			
f	966				8 0 0	974	0.1
m	994				7 3 3	1019	0.2

From the fact that of C, Si, Ge and Sn we know modifications with the same structure as diamond, while this is not the case for Ti, Zr and Th, we might conclude that C and Si are somewhat more intimately connected with the elements of group IV*b* than with those of group IV*a*.

To Prof. Dr. W. H. KEESOM I am much indebted for his interest and his kind help in this investigation.

[1]) In the calculation of the intensities, only the structure factor, the LORENTZ-factor and the number of planes factor have been used, not the polarisation factor and the temperature factor.

Physiology. — "*An Objective Method for determining the Coagulation-time of Blood.*" By R. J. Wolvius. (Communicated by Prof. A. A. Hijmans van den Bergh.)

(Communicated at the meeting of December 23, 1921).

The usual methods for determining the coagulation-time of blood aim at detecting the right moment at which the phase of complete solidification of the blood has just set in.

At first I myself adopted the method suggested by Fonio and Frank, viz. by observing, with strict precaution, the coagulation of the blood on a watchglass and by noting down the moment at which the phase of complete solidification had apparently been reached. However I was always in doubt whether complete solidification had been accomplished at a certain moment, or whether it had not, so that I always hesitated in fixing the right moment.

In this connection Hayem[1]) says: "On sait, en effet, que la solidification du sang ne se fait pas brusquement, c'est à dire d'un seul coup, à un moment précis. Le phenomène, évolue d'une manière progressive, à tel point, que pendant une periode relativement assez longue, on reste dans l'hésitation, en se demandant si la prise en gelée est effectuée ou n'est encore qu'imminente."

What tells most against these methods, is that the degree of solidification has to be determined by subjective observation. I, therefore, looked for some phenomenon that goes on pari passu with the solidification and admits directly of measurement. I found that phenomenon in the turbidity which attends the salting out of fibrin and consequently decided to measure it. Preliminary experiments had shown that at the very outset of thickening of the blood a clouding commences that increases with the further progress of the thickening and ultimately remains stable as soon as coagulation has reached its completion. Now, it being my purpose to observe the time in relation to solidification, I might as well ascertain the time taken up by the clouding process. In order to measure this growing turbidity I made use of a new apparatus, the extinction

[1]) Hayem, Du Sang, quoted from Marcel Bloch, La coagulabilité sanguine pag. 22. Thèse, Paris 1914.

meter[1]) of Dr. W. J. H. Moll, which enables us to measure the turbidity from moment to moment.

The principle of this apparatus may be discussed in a few words:

A powerful light-source is firmly set up between two surface thermobatteries I and II, after Moll. They are both connected to a mirror-galvanometer, thus counteracting each other. Between the lamp and the thermobattery I is placed a cuvette filled with water; between the lamp and the thermo-battery II a cuvette filled with the blood-plasm.

Consequently the light that is directed on to the thermo-batteries is weakened on the one side by water, and on the other by blood-plasm. Through displacement of one of the thermo-batteries or through changing the position of the lamp the unevenly weakened light may be made to fall upon the thermo-batteries with equal force. The two thermo-electric currents thus elicited, will then be equal, the galvanometer will receive a current 0, the image reflected by the mirror will occupy the 0-position. The apparatus has then been "adjusted". The slightest change in the turbidity of the plasm disturbs the equilibrium and yields a deflection of the galvanometer; the apparatus acts so quickly that after a contingent sudden change in the turbidity the reflected image will come to rest again within a few seconds. Moreover a procentic measurement may be taken of the changed turbidity with the aid of a so-called compensation-switch.

Now our procedure is as follows: Into a sterile, dry Record-syringe of 10 c.c. with a sharp, dry needle, 1 c.c. of a clear sterile solution of 1 % potassium oxalate in 0.85 % common salt is sucked up; the needle is inserted into a cubital vein and the blood is aspirated to 10 c.c. Due regard should be given to an easy flow of the blood into the syringe, so that no air is drawn in along with it. The mixture thus obtained, is centrifugalized during 20 minutes in sterile centrifugation-tubes, which causes the blood-corpuscles and the blood-platelets to precipitate and the supernatant, more or less turbid plasm can be pipetted off and transmitted to sterile tubes. Three c.c. of this oxalate-plasm (measured very carefully with sterile pipettes) are put into pure, and dry cuvettes. The cuvettes used by me are made of the same glass and have precisely the same dimensions, so that not only the thickness of the fluid-layer, but also the level to which the cuvette is filled, is always the same in every one of them; in other words the contact-plane

[1]) Dr. W. J. H. Moll, Een extinctiemeter. Verslag Koninklijke Akademie van Wetenschappen, Wis- en Natuurkundige Afdeeling, 27 Maart 1920. Deel XXVIII.

between plasma and glass is the same. This cuvette is placed in another one of special construction, which acts as a thermostat, is filled with water and is heated electrically.

The measurement proceeds as follows:

The cuvette is placed in the thermostat so as to make the light of the lamp reach the thermo-battery II and pass through the plasm. By its side, in the same thermostat, stands a test-tube containing $1\frac{1}{2}$ c.c. $^1/_2\,^0/_0$ $CaCl_2$. The extinctionmeter is "adjusted". Then follows a 20 minutes' wait, after which the plasm and the $CaCl_2$ will be of the temperature of the thermostat and the galvanometer will be completely quiescent and in the zero-position. The work-room is made semi-dark and from this moment photographical readings are taken from the galvanometer. A registering instrument is used that is moved by a perfectly reliable clockwork.

After some moments the $1\frac{1}{2}$ c.c. $^1/_2\,^0/_0$ $CaCl_2$ are added to the plasm, the whole mixture is rapidly stirred for half a minute with a sterile glass rod and is then left to itself. The galvanometer then traces on the sensitive paper of the registering instrument the "turbidity-curve".

Fig. 1 is a faithful reproduction (natural size) of such a curve, in

Fig 1.

which three horizontal portions *A*, *B*, and *C* are to be distinguished:

A indicates the motionless phase of the galvanometer during the time when there is only the oxalate-plasm between light-source and thermo-battery; *a.* indicates the moment when $CaCl_2$ is added; the first moment the plasm becomes clearer, see *b.*, which is owing to the dilution of the plasm. But directly after this the liquid becomes very turbid which is due to the forming calciumoxalate; the curve ascends almost vertically, see *c.*, and would gradually have reached the *B*-level, if not an irregular weakening of the light had been brought about by the stirring rod; the jerks in the curve at *d.* illustrate this irregularity, so that they have nothing to do with the process of turbidity. The calcium oxalate is not precipitated but remains in suspension; for a few minutes the galvanometer remains constant, as shown in the *B*-portion.

Up to this moment coagulation is out of the question. Solidification commences at *e.* at a certain moment, apparently quite independently from the inital turbidity (formation of calcium-oxalate), and simultaneously the second phase of turbidity begins. It is this portion of the curve that interests us most. Its shape is an objective illustration of the coagulation process.

It appears that this coagulation starts very slowly, then proceeds more quickly until a maximum rate is attained, after which a retardation sets in again until the terminal value is ultimately reached.

I will not discuss here the nature of this curve, but only point to the method, which enabled me to typify any given portion of the curve by a figure. I note the exact time at which certain levels, e.g. $^1/_4$ and $^3/_4$ of the difference of the *B*-, and *C*-level are reached; the time-difference is my control-number.

My researches were performed in the Physical Laboratory of the Utrecht State-University, where I had the freedom of the instruments.

Utrecht, December 1921.

Physiology. — "*A contribution to the physiology of the electrical organ of Torpedo*" By Prof. F. J. J. Buytendijk. (Communicated by Prof. G. van Rijnberk).

(Communicated at the meeting of April 29, 1922).

In the winter of 1911 I had the opportunity to investigate the function of the electrical organ of Torpedo in the Zoological Station of Naples. The aim of part of this research was to study the magnitude and character of its discharges under different circumstances. For this purpose a string-galvanometer (large type of Edelmann) was available at Naples and with this apparatus I made many records. From these records and from test-records of the apparatus it appeared, that the string-galvanometer is not the most suitable instrument for the registration of the discharges of the organ of Torpedo which reach their maximum within 0,002—0,003 sec. This does not astonish us in connection with the investigations of Garten [1]). For this reason I intended to continue this research with an apparatus more suitable for this purpose (oscillographion, Fuji [2]), string-electrometer, Cremer [3]). However, as circumstances prevented me from carrying out this plan, I now communicate the results of my research.

Marey, Schönlein and Gotch [4]) have already observed that the reflex-discharge of the electric ray has a rhythmical character. In many records I found that as a rule many discharges succeed each other,

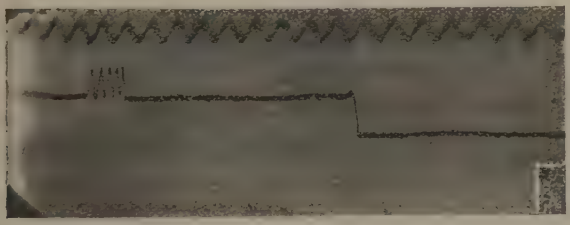

Fig. 1. Reflex-discharge in Torpedo after mechanical stimulation. Test-record 4 volt. Time $^1/_{50}$ sec.

[1]) Garten. Abh. d. Kgl. Sächs. Ges. d. Wiss. 1899.
[2]) Fuji. Journ. of the College of Science Univers. Tokio 1914, Vol. 37.
[3]) Cremer. Sitz. Ber. Physiol. Geselsch. Berlin 1912. Mediz. Klinik 1912, N°. 42.
[4]) S. Garten. Handb. d. Vergl. Physiol. 3e Bd. 2e H., p. 177.

mostly 5—8 in number with an interval of 5 σ. Fig. 1 represents one such discharge, where the shocks came with an interval of 5,6 σ. Cremer found an average of 5,6 σ in reflex-discharge. Most striking in these rhythmical discharges is the regularity and the equal amplitude of the single shocks, as appears clearly from my figure. In the periodical discharges after stimulation of the nerve of the excised organ, I never obtained such regularity and as a rule inequal amplitude. Usually the shocks diminish gradually, sometimes they first increase, then decrease. Fig. 5*a* and 5*b* illustrate this more clearly. The periodical discharges in reflex-action therefore give the impression of being caused by a series of central impulses from my nervous system, whereas the periodical discharges after stimulation of the organ or the nerve seem to be due to secondary self-stimulation. This is especially and to a greater extent true in the case of the stimulation of the nerve. After direct stimulation of the organ usually only two small, secondary discharges occur, provided that the nerve has been cut at the very spot of its entrance into the organ. Fuji however has registered reflex-discharges (in Astrape japonica) in which only two shocks occurred in every group followed by a small one. The same result was also obtained after stimulation of the nerve stem. For this reason Fuji believes that the successive discharges occur by self-stimulation. The solution of this question has importance for the question which has been solved by Garten [1]) in Malapterurus, i. e. in how far the discharges of both organs occur simultaneously.

In a detailed research Bernstein and Tschermak [2]) have tried to find out whether the current which the electrical organ produces during activity is caused by a concentration-chain or whether a so-called chemical chain here causes the difference in potential. To solve this question the temperature-coefficient of the force of the current in the organ was investigated within certain limits of temperature.

From theoretical considerations it is known, that in a concentration-chain the E. M. F. is nearly proportional to the absolute temperature. Bernstein already had found a positive temperature-coefficient for the current in muscles and nerves and within normal limits of temperature the E. M. F. proved to be nearly proportional to the absolute temperature.

In their study on the electrical organ the authors mentioned above

[1]) Garten. Zeitschr. f. Biol. 1910. Bd. 54. S. 399—430.

[2]) Bernstein und Tschermak. Pflügers Archiv. 1906, p. 112.

always stimulated the nerves of the organ by means of a single induction-shock (make-induction) and read the deviation of a galvanometer with not too great inertia on a scale. The deviations of such galvanometers are nearly proportional to the average E. M. F. of short currents, if the external resistance and the path of the current remain constant. The latter condition, however, was surely not complied with at different temperatures.

From the investigations of Gotch on the capillary electrometer it had already become known that the velocity of the movement at low temperature (3° C.) is retarded and differs rather strongly from that at moderate temperature (15—20° C.). Such change in the record of the current must magnify the movements at low temperature. In that way the difference with those found at higher temperature is bound to become smaller than corresponds to reality and the observed temperature-coefficients must be too small as Garten has already observed. Moreover, it has become evident from the experiments of Schönbein and Garten, that after indirect stimulation of the electrical organ by induction-shocks a repeated discharge frequently occurs, which of course does not show up with the slow galvanometer.

My own experience has shown that especially in the cooled organ this repeated discharge occurs very frequently. In this way the very low temperature-coefficients, found by Berenstein and Tschermak may partly be explained.

At any rate, it seemed advisable to try to secure more data on the process and the E.M.F. of the shock. Of freshly caught specimens of Torpedo marmorata and T. occalata the electrical organ was prepared free with its nerves after removal of the skin. The organ was now enclosed between two zinc electrodes by means of two rubber rings. Two bars for the conduction of the electrical shock were attached to the zinc electrodes.

The conductors were passed through a rubber stopper which also held the platinum electrodes used for the stimulation. This rubber stopper served to close a glass-vessel, in which the organ could be enclosed and through which a stream of liquid could be passed. The strength of the electrical discharge could thus be studied under different circumstances.

Moreover, a thermometer was inserted into the stopper. If one allows the liquid to flow into the vessel through the lower tube (Fig. 2) and to leave the apparatus by the upper one, it is possible to regulate conditions so as to keep the organ in the fluid and to keep the nerves in the air.

This arrangement had the advantage that it enabled me to repeat the indirect stimulation under very constant conditions. In this way one succeeds in keeping the organ in good condition for 2—3 hours,

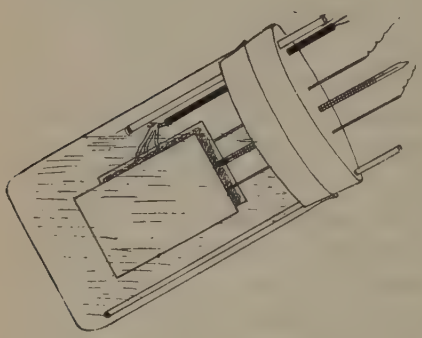

Fig. 2. Apparatus for study of electrical discharge in different fluids, gases and temperature.

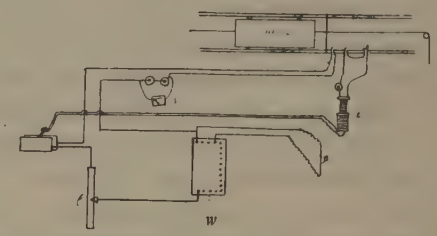

Fig. 3.
lo = electr. organ
l = slate-resistance
v = volt-meter
w = resistance-box
s = string-galv.m.
i = induction-app.
ph$_2$ = apparatus for photogr. registrat.

so that if one stimulates once every 15 minutes the deviations of the galvanometer remain constant.

The discharge was led to an ordinary resistance-box and a slate-resistance (of 800,000 Ω). From the resistance-box a circuit could be branched off to the string-galvanometer. By means of a key, connected with the recording apparatus, a definite potential difference could be thrown into the chief circuit for the purpose of testing the movements of the string (Fig. 3).

A tuning fork of 50 vibrations per second marked the time on the photographical plate, while a very sensitive signal of Deprez indicated the moment of stimulation. The nerves were stimulated by means of induction-shocks. Sometimes part of it was thrown into the string-circuit so as to have the string itself mark the moment of stimulation.

It is clear that this method does not enable one to study the question of the relation of a stimulus to its effect.

This question has been thoroughly investigated by Fuji with the oscillographion.

In my experiments I could not state anything but the fact that a weaker stimulus gave a less noticeable effect than a strong one and

that make induction-shocks are more effective than break-shocks (Fig. 4).

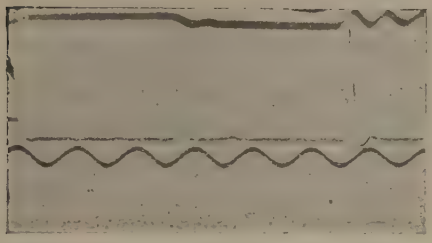

Fig. 4. Example of discharge after indirect nerve stimulation with make and break induction shock.

To get an impression of the conditions on which the magnitude of the discharge depends, I first investigated the question whether organs which had been kept in different liquids for some hours, as a result showed a change of the discharge-shock. Of course, the nerves were always stimulated with maximal stimuli.

In this way it appeared that an organ (without skin) kept in:

Experiment 1.

2,5 % NaCl-solution lost its irritability after three hours. The same thing was true for sea water. In Führer's solution + urea [1]) the irritability strongly diminished after three hours.

Experiment 2.

Organ 1: in (NaCl 2,5 % + KCl 0,1 %) no shock could be obtained after 40 min.

Organ 2: in (NaCl 2,5 % + $CaCl_2$ 0,2 %). After 40 min. the shock had diminished slightly.

Organ 3: in F. sol. after 40 min. shock not changed.

Experiment 3 (see fig. 5).

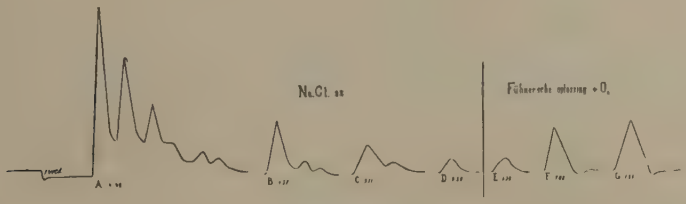

Fig. 5.

A preparation made from Torpedo marmorata (size 15 c.m.) from 3.50—4.10, Organ 1 is put into NaCl 3 %. From 3.50—5.28 four records were made (fig. 3. *a, b, c, d*), the preparation was then put into Führer's solution + O_2. After about 60 min. record *e*, after another 38 min. record *f*, after 20 min. record *g*.

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[1]) Führer Zeitschr. f. allgem. Physiol. (1908) Bd. 85. 485.
Used solution was:

Na_2CO_3	0.2	per 1 L. water.
$CaCl_2$	0.2	
KCl	0.1	
NaCl	20.	
Urea	25.	

It appears that after the discharge has decreased in the NaCl solution, it again increases in Führner's solution + O_2.

The other organ (2) has been kept in moist air from 4.10 until 6.47, then it is put into Führner's solution + O_2. It becomes apparent that this organ also shows a considerable increase in magnitude of the discharge after 30 minutes.

The result of experiment 3 is therefore that O_2 causes the shock to increase and that sol. F. shows the same activity even though the discharge had been weakened by an immersion in NaCl-solution.

Experiment 4 (Fig. 6 and Fig. 7).

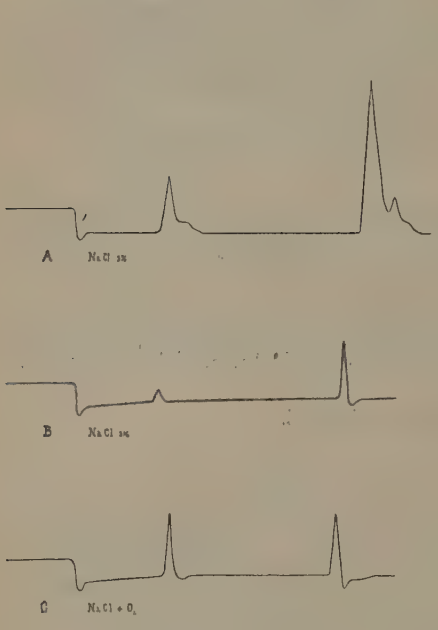

Fig 6.

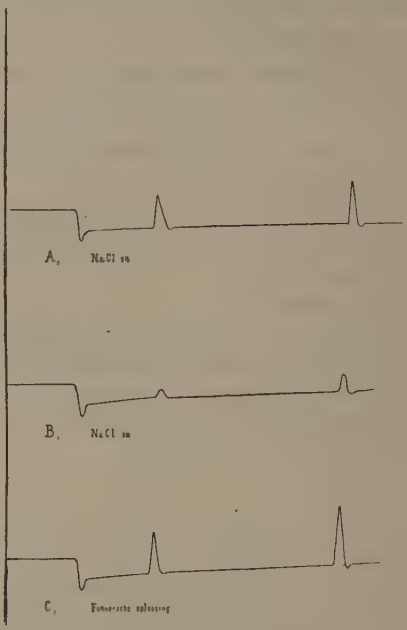

Fig. 7.

Preparation of small Torpedo (15 c.m.).

Organ 1 (Fig. 6): At 3.53 in 3 % NaCl. Record *A*. After two hours record *B*. O_2 is now bubbled through the NaCl. After 30 min. record *C*.

Organ 2 (Fig. 7): At 4.50 in 3 % NaCl. Record A_1. After 70 min. record B_1. NaCl replaced by Führner's solution. After 30 min. record C_1.

Result: O_2 restores the discharge even if the organ remains in NaCl-solution. Sol. F. restores the discharge which has been diminished in NaCl. solution.

Experiment 5.

The preparations were made at 10.30 in the morning.

Organ 1: kept for 4 hours 40 min. in F. sol. poor in oxygen (boiled).

Record *a*: deviation is 0,25 volt. After 14 min. exposure to the air.

Record *b*: deviation is 0·31 volt. After the preparation had been kept for 23 min. in well-aerated F. sol.

Record *c*: deviation is 1,4 volt.

Organ 2: kept for 4 hours 30 min. in F. sol. (O_2 bubbling through).

Record a: deviation is 3 volt (followed by a series on spontaneous discharges). In F. sol. free of O_2 the deviation diminishes considerably. After the preparation has been kept for a long time in F. sol. rich in O_2, a repeated discharge follows after one indirect stimulation.

In two other experiments in duplicate it could be shown that also after keeping the organ in F, sol. which had been saturated with H_2 and more so yet in one with CO_2, O_2 causes the deviation to increase.

After this series of experiments another one followed in which the same apparatus was used (Fig. 2). The Fühner's solution, poor in O_2 or saturated with, it was here kept at certain definite temperatures by immersing the whole apparatus in a thermostat.

Experiment 8.

Organ 1: Temp. 18°. Deviation about 25 volt. Latent period 5,5σ.
After 30 min. Temp 11°. Deviation 24,2 volt. Latent period 6,8σ.
After 22 min. Temp. 28°. Deviation 0,3 volt. Latent period 4,2σ.
Organ 2: Temp. 18°. Deviation 20,6 volt. Latent period 6,4σ.
After 40 minutes: Temp. 30°. Deviation 0,25 volt. Latent period 3,4σ.
After 26 minutes: Temp. 15°. Deviation >4 volt. Latent period 6,5σ.

Experiment 9.

Piece of organ of large Torpedo between zinc electrodes, two nerves being intact. Thermometer lies in immediate neighbourhood of organ. Length of nerves 15 m m. Distance of stimulating electrodes 3 m.m.

Temp. degrees C.	Time. Hour.	No.	Test.	Test deviation m.m.	Discharge Volt.	Lat. per. in σ.
20	5.05	*a*	4 volt	5	>40	5.2
13	5.15	*b*	4 volt	5	± 30	7.5
7.5	5.37	*c*	4 volt	5	32	12
6	5.53	*d*	4 volt	5	10.5	20
5	6.32	*e*	$^1/_2$ volt	5.5	0.27	22.3
10	6.47	*f*	$^1/_{16}$ volt	9	0.31	16
15	7.16	*g*	$^1/_2$ volt	5.5	0.67	8.7
15	9	*h*	$^1/_4$ volt	14	0	

As mentioned before we can not attribute any absolute value to the volt-values given for the discharges, since the galvanometer-records require a correction which can not be calculated very easily. After cooling, however, the string will more easily follow the potential difference, because it develops more slowly in the cold.

Whatever may be their absolute value we can see easily from

figures of exp. 9 that as long the organ is cooled, up to about 7,5°, the discharge diminishes slightly, though not very considerably. From 5—15° a rapid decrease follows with a slight recovery. Then the organ apparently is dying.

The changes in deviation of the test-potential were obtained by changing the side-chain or the resistance, not by changing the sensitiveness of the string (tension).

Experiment 10.
Large organ, kept in F. sol. containing O_2.

Time.	Temp.	Test. volt	Deviation Test. m.m.	Discharge Volt	Latent Period in σ
4.23	5	1	11.5	0.33	12.26
4.38	10	1	14	0.49	9.33
5.53	19	1	7.5	3.3	5.6
5.14	26	1	7.5	0.67	4.66

The temperatures given are presumably not those of the inside of the organ because the temperature changed too rapidly. Consequently the large organ could not have assumed the temperature of the environment.

Heating to above 22° gives a considerable decrease as appears from:

Experiment 11.
At 3.05 'o clock. Temp. 21°. 1 volt = 2,6 m.m. Deviation 25 m.m.
At 3.30 'o clock. Temp. 28°. 1 volt = 2,6 m.m. Deviation 2,1 m.m.
The rise in temperature has diminished the deviation to $^1/_{10}$.
Experiment 14.
At 5 'o clock. Temp. 20°. 1 volt = 5,5 m.m. Deviation 32 m.m.
At 5.30 'o clock. Temp. 25°. 1 volt = 5 m.m. Deviation 17 m.m.
The rise in temperature has caused the deviation to diminish to $^1/_2$.

In Fig. 8 and 9 the string-record of the discharge at different temperatures has been pictured together with the test-record. I have reconstructed it to the best of my ability from the not absolutely focussed photographs.

Large organ in F. opl. at 28° C. (since 12.55).

Time.	Test. 1 volt	Deviation.
1 11	5 m.m.	35.5 m.m.
1 27	5 m.m.	34 m.m.
Now 8 stimuli are given at intervals of 30 sec.		
1.31	5 m.m.	4.8 m.m.

In that way it is evident that at high temperatures fatigue occurs very rapidly. A attempt to study the influence of O_2 at different temperatures could not be carried out systematically.

Fig. 8.

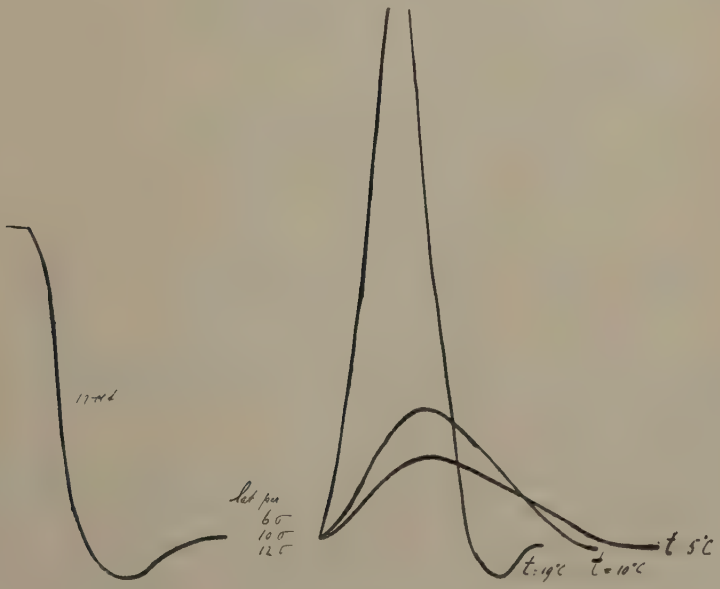

Fig. 9.

From the experiments it appeared however that at high temperatures (22°—28°) O_2 did not cause an increase in discharge. The latter was the case at lower temperatures. Experiments on the influence of narcotics had to be given up at an inopportune moment.

Fig. 10 demonstrates that by chloral-hydrate the discharge disappears nearly completely.

In the apparatus nerve and organ were kept at the same temperature. It seemed important in order to judge about the change

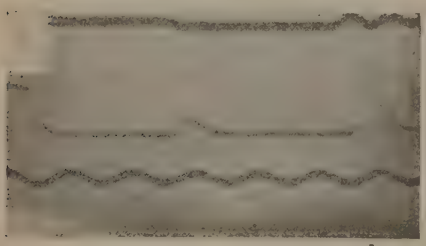

Fig. 10*a*.

a. Record of discharge small electrical organ). Test $^1/_4$ volt.

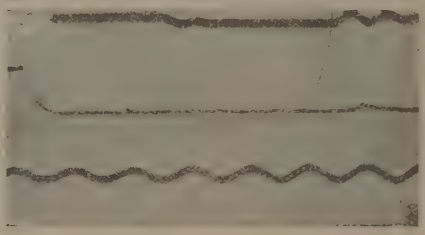

Fig. 10*b*.

b. The same after exposure to chloral-hydrate dissolved in Fühner's solution.

in the values for the latent period, to submit the nerves separately to changes in temperature.

Experiment 16.

Preparation of large Torpedo (28 c.m.) made between 11.45 and 12.45 and put into apparatus.

Piece of organ with two nerves, the nerves led through glass tubes, in which stimulating electrodes. Tubes surrounded by glass-mantle, in which water circulates at different temperatures.

After the experiment has been ended a control is made by tying off the nerves, which causes a complete breaking of the conduction.

The magnitude of the deviation remained the same although the temperature of the nerve varied from 20'—6°.

The values of the latent periods actually were lower at lower temperatures. The measurements are, however, not sufficiently accurate to allow a calculation of the velocity of the conduction. That, however, the differences in latent period found in the other experiments are due to changes in the electrical organ, is evident from the fact, that in this experiment the difference between 6°—20° only amounted to 1.8 σ.

Having thus obtained an impression on the influence of temperature O_2 and different salt-solutions on the strength of the discharge as a responce to indirect stimulation. I have tried to study the gaseous exchange of the electrical organ in rest and during activity.

For this purpose the electrical organ was enclosed in a very thin

dialysing sac of collodion. The oxygen of the surrounding liquid passes through these sacs without difficulty, as could be shown in preliminary experiments, but only a very small quantity of organic substances passed through so that the method of WINKLER for the determination of O_2 dissolved in water, could be used with a slight correction. This very useful method of WINKLER can only be used in the absence of organic substances in the liquid or after a correction has been made [1]).

This dialysing sac was put into a bottle, which was completely filled with FÜHNER's solution of known O_2 content. After some time (1—2 hours) a certain quantity of the contents of the bottle was again secured for a WINKLER's determination of its O_2 percentage.

In order to stimulate the organ during its stay in the liquid, two silver electrodes were tied to the organ. Organ and electrodes were then put into the dialysing sac and immersed into the liquid. The organ was stimulated directly once every 5 sec.

The results are given in the following table:

TABLE I.

Exp. N°.	Weight organ in gms	Stim. + Not stim. —	Time of resp. in hours.	Temp. ° C.	O_2 consumed in cc. $^1/_{100}$ N thio per hour and K.G.	
					Not stimul.	Stimul.
6	35	—	2	18	60	
7	36	—	1 1/4	18	62	
7a	36	+	1/2	18		103
8	41	—	1	21	220	
8a	41	+	1	19		220
9	57	—	1	21	103	
9a	57	—	1	19	74	
10	18.3	—	1	21	306	
10a	18.3	+	1	21		440
12	28	—	1	6	136	
12a	28	+	1	6		160
13	41	—	1	19	93	
13a	41	+	1	19		100

[1]) See HENZE Abderh. Handb. der Biochem. Arbeitsmeth. III_2 p. 1067.

In the first place it became evident that though the results varied considerably — this is easily explained by the changing condition of weight etc. of the organ and the varying conditions for the diffusion of oxygen — the figures in the first place give a very definite idea about the gaseous exchange in the electrical organ. The electrical organ appears to consume 6—18 m.m³. O_2 per gram-hour, a quantity which is of the order of the O_2 consumption of the peripheral nervous system (THUNBERG) [1]) which consumes about $^1/_{10}$ of the quantity that is used by the central nervous system.

In 1883 WEYL tried to demonstrate some chemical changes in the electrical organ after it had shown vigorous activity. He found a change in reaction (hydrogen-ion concentration) i.e. an increase in acidity after activity. Moreover, he tried to estimate the production of CO_2 by the organ. WEYL found that 17,5 gms. produced 4 mgms. CO_2 in two hours. After stimulation he found a decrease instead of an increase. The alcohol-extracts of a stimulated organ and one which had not been stimulated, did not show any differences. The watery extract of the stimulated organ was larger. I myself have tried in vain to demonstrate the change in reaction after stimulation. I have tried to demonstrate two chemical substances in the electrical organ i.e. xanthin-bases and glycogen. This seemed to me to be of importance, because we know that the electrical organ must be derived from muscles in which both substances occur abundantly.

The xanthin-bases were determined according to the method of BURIAN:

100 gr. of the organ are boiled for 12 hours in 1 L. of 0,5 % H_2SO_4. After filtration the sulfuric acid is precipitated with $Ba(OH)_2$ and the liquid which is now alcaline is filtered. The filtrate is saturated with CO_2. The $BaCO$ is removed by filtering and the filtrate, after acidification with acetic acid, is evaporated down to 100 c.c. These 100 c.c. are boiled for some time with a smaller quantity of concentrated $NaOH + Na_2CO_3$ and filtered. The filtrate is acidified with HCl. A precipitate of xanthin bases now comes out on addition of an excess of $NH_3 + AgNO_3$. In this precipitate nitrogen can be determined according to KJEHLDAHL.

The following table gives the results of this investigation: (See Table 2, following page).

We may therefore conclude that the electrical organ contains no xanthin-bases or a neglegible quantity.

Determinations of glycogen were made in two very large animals and in two young ones.

[1]) THUNBERG. Zbl. Physiol. 28 (1904). See also BUYTENDIJK. Kon. Ak. v. Wetensch. 1910 (615—621).

TABLE 2.

		Result.
Exp. 1.	107,5 gm. electric organ.	Very slight precip.
Exp. 1a.	55 gm. muscle	Heavy precip.
Exp. 2.	106 gm. electric organ.	No precip.
Exp. 3a.	120 gm. electric organ.	Very slight precip.
Exp. 4.	122 gm. electric organ.	No precip.
Exp. 4a.	52 gm. muscle	Heavy precip.

The determination was made according to PFLÜGER:

TABLE 3.

Exp.	Size of animal in c.m.	Quantity of organ.	% Glycogen.
1	60 taken out of the aquarium.	313	0.051
2	45 taken out of the aquarium.	128.8	0.031
3	15 freshly caught animal.	20	0.787
4	5 6 specim.; just born.	13.5	1.02

If we compare these results with those of BAGLIONI [1]), we see that the glycogen values in exp. 1 and 2 are lower than those given by BAGLIONI (0,09 %), but of the same order. The figures in very young animals (exp. 3 and 4) are very much higher.

If the electrical organ functions by splitting up ion-proteids, we may expect salts to become free. This might be the explanation of the increase of the watery extract found by WEYL. The electrical organ reacts to mechanical stimulation, e.g. mincing or pressure, by strong activity. Therefore I have tried to divide electrical organs finely by gradually cooling and finally freezing them. In that way no discharges occurred. In centrifuging the frozen organ after it had been ground up, I could obtain a very complete separation of the organ-fluid. This fluid as obtained from stimulated and fresh organs, was used for the present research.

In the liquid thus obtained I made a determination of the ash-compounds, which led to the following results:

[1]) BAGLIONI. Hofmeister's Beiträge 1906. Bd. VIII, p. 456—471.

TABLE 4.

Ash (mgs per c.c. organ-fluid) determined after the wet method (SO_4 ash).

Exp.	Not stimul.	Stimul.	Difference.
1	31.8	31	— 0.8
2	30.4	31.2	+ 0.8
3	38.6	35.5	— 3.1
4	26	28	+ 2

We see at once that there is no difference in ash-content in the fluid of a stimulated organ and of one which had not been stimulated. Moreover I made freezing-point determinations in both liquids.

TABLE 5.

Δ in ° in organ-fluid.

Exp.	Not stimul.	Stimul.	Difference.
1	2.12	2.215	+ 0.095
	2.12	2.21	+ 0.09
2	2.145	2.30	+ 0.155
	2.135	2.295	+ 0 160
3	2.16	2.23	+ 0.07
	2.15	2.24	+ 0.10
4	2.095	2.18	+ 0.085
	Ground when frozen.	Ground when warm (20°).	
5	2.08	2.25	0.17

From these experiments it appears that during activity substances pass into the organ-fluid which must be considered to be organic substance, because the ash-content is not increased, whereas the freezing point shows a very definite lowering.

However incomplete these investigations may be, I have felt the desirability of communicating them very briefly, the more because I shall most probably not be in a position to take up the whole problem once more and because the data published in the present paper, to my opinion, may be a stimulus to a continued research of the physical and chemical processes which take place during the discharge of the electrical organ.

Physiology. — *"On the Significance of Calcium- and Potassium-ions for the artificial Oedema and for the lumen of the bloodvessels"*. By RUDOLF J. HAMBURGER. (Communicated by Prof. H. J. HAMBURGER).

(Communicated at the meeting of March 25, 1922).

Of late years a series of researches have been carried out in the Groningen Physiological Laboratory, which demonstrated, among other things, that the solution used by SIDNEY RINGER in his remarkable experiments on the frog's heart, is not suitable for other organs of the frog. As known, this solution is composed as follows: NaCl 0,7 %, $NaHCO_3$ 0,02 %, $CaCl_2 \cdot 6$ aq. 0,04 %, and KCl 0,01 %. HAMBURGER and BRINKMAN [1]) found that when, after the addition of a little glucose, this solution is allowed to perfuse the kidney, the glomerular epithelium does not retain any glucose. Systematical investigation, however, enabled them to modify the circulating fluid in such a way that the glomerular epithelium obtained the property to retain the physiological amount of glucose (0,060—07%) This circulating fluid was of the following composition: NaCl 0,5 %, $NaHCO_3$ 0,2—0,285 %, $CaCl_2 \cdot 6$ aq 0,04 %, KCl 0,01 %. Also for other organs of the frog the proper physiological fluid was found after systematical investigation (for the movements of the stomach on excitation of the N. vagus [2]), for the movements of the rectum [3]), for formation and solution of biliary concrements) [4]). All these researches have shown that the efficiency of the circulating fluid depends on the amount of free calcium-ions [5]). Also in warm-blooded animals the concentration of Ca-ions appeared to play a prominent part: here we allude to the investigations on haemolysis [6]) and on

[1]) H. J. HAMBURGER and R. BRINKMAN, These Proceedings Vol. XIX, p. 989 and Vol. XX, p, 668. See also Biochem. Zeitschr. 88, 97, 1918.

[2]) BRINKMAN and VAN DAM, These Proc. 18 Dec. 1920.

[3]) Demonstration by VAN CREVELD at the Conference of Physiologists at Amst. 22 Dec. 1921,

[4]) BOLT and HEERES, Ned. Tijdschr. v. Geneesk. 65, 2d half N°. 10, 1921. Also Pflüger's Archiv f. d. ges. Physiol. (not out yet).

[5]) Cf. also H. J. HAMBURGER, Permeability in Physiology and Pathology, Lancet 2, 1055, 1921.

[6]) BRINKMAN, Biochem. Zeitschr. 95, 101, 1919.

the origin of spasmophilic phenomena consequent on decrease of the concentration Ca-ions of the blood [1]). This concentration of depends upon Ca-ions the $NaHCO_3$ and on the H-ion concentration.

As for the influence of the concentration of the Ca-ions upon the permeability of the glomerular epithelium, it is so great that, even when a *potassium-free* liquid is sent through the kidney, retention of the physiological quantity of glucose was still observable [2]).

This being the fact, it seemed interesting to us to investigate *with what liquid the vascular system of the frog had to be perfused in order to prevent the production of œdema in the hindlimb.*

We were all the more induced to inquire into this matter, since some years back GUNZBURG [3]) occupied himself with this question in the Utrecht Physiological Laboratory. He found that, when perfusing the vascular system of the frog with a fluid such as RINGER had used for the heart, and which differed from ours in $NaHCO_3$ 0.02 % being used instead of 0,2 %—0,285 %, KCl 0,01 % was indispensable to prevent oedema. So, in GUNZBURG's experiments oedema arose when the fluid was potassium-free or when too large an amount of K was present. Instead of K he could use also Uranium, Thorium or Rubidium in definite quantities. It is evident, therefore, that, according to GUNZBURG, K is indispensable in this case, and this indispensability is, according to him, due to the specifically radio-active effect of this element.

But GUNZBURG also detected that in RINGER's mixture K could be left out, when the mixture was saturated with oxygen, in which case œdema was also prevented. We shall revert to this point.

It has been stated that in the circulating fluid HAMBURGER and BRINKMAN could do entirely without K. In that case, however, the Ca-ion concentration should have a definite value. The question now arose: *can œdema be prevented in the frog's limb with a potassium-free circulating fluid, the Ca-ions concentration being accurately fixed?*

In order to find an answer to this question we have a series of experiments which yielded unexpected results with reference to the influence of the Ca-ions concentration on the lumen of the blood-vessels (capillaries).

Of course the inquiry was begun by repeating GUNZBURG's experiments. A perfusion of the ordinary RINGER's liquid ($NaHCO_3$ 0,02%)

[1]) VAN PAASSEN, Ned. Tijdschr. v. Geneesk. **65**, 2e helft, nr. 17, 1921.

[2]) HAMBURGER and BRINKMAN, These Proceedings Vol. XX, p. 668; Biochem. Zeitschr. **88**, 97, 1918.

[3]) GUNZBURG, Arch. Néerl. d. Physiol. **2**, 364, 1918.

did not cause œdema, which indeed came forth, when K was left out, just as GUNZBURG has shown.

Now mixtures were applied of NaCl 0,6 % with several quanta of $CaCl_2$. 6 aq. The result was that œdema appeared when $CaCl_2$. 6 aq. 0,003 %, 0,005 %, 0,006 % was applied, but that *it stayed away when* $CaCl_2$. 6 *aq.* 0,007 % was used, even when the hydrostatic pressure was raised from 35 to 70 cm.

These experiments, therefore, went to show that, contrary to GUNZBURG's opinion, K may be taken from the circulating fluid, without evoking œdema [1]), in other words, that *no radioactive substance is required* to prevent œdema.

Now it seemed to be interesting to add some K to this circulating fluid (NaCl 0,6 % + $CaCl_2$. 6 aq. 0,007 %), which, as has been said, does not cause œdema. The addition of KCl 0,01 % produced œdema. It may be concluded, therefore, that *the absence of œdema on adding KCl in the experiments of* GUNZBURG *cannot be due to a specific Potassium-action.* On the other hand, it became rather evident, that when a definite concentration of it is present in our NaCl—$CaCl_2$ mixture, œdema is sure to appear on that account, so that it becomes obvious, that the prevention of œdema by the 0,007 % $CaCl_2$ 6 aq. solution is balanced, and even more than balanced, (this depends on the amount) by the antagonistic Potassium.

Now, how are we to account for GUNZBURG's finding that a potassium-free RINGER's mixture evokes œdema? It is probably to be ascribed to the fact that this circulating fluid contained only 0,02% of $NaHCO_3$, of which according to RONA and TAKAHASHI's [2]) formula a large amount of Ca-ions is the consequence.

A direct measurement of the Ca-ions concentration after BRINKMAN and VAN DAM [3]) proved that the Ca-ions concentration used by GUNZBURG was, indeed, much greater than in the above-named mixture of NaCl 0,6 % + $CaCl_2$. 6 aq. 0,007 %.

We found experimentally that in our mixture NaCl-$CaCl_2$, which did not bring about œdema, contained 13 mgrms of Ca-ions per Litre, whereas the liquid used by GUNZBURG contained 20 mgrs per Litre, which makes a difference of 35 %.

Now we know that K and Ca are antagonists; the action of K being liquifying, that of Ca tending towards coagulation. It is not

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[1]) It has been shown that, with other kinds of frogs, there are sometimes other Ca-ion concentrations needed to prevent œdema.

[2]) RONA u. TAKAHASHI, Biochem. Zeitschr. 49, 370, 1913.

[3]) BRINKMAN and VAN DAM, These Proceedings, Vol. XXII, p. 762.

surprising, therefore, that the ratio of K- en Ca-ions is of great importance for the permeability of the vascular wall. It is evident that in an NaCl-$CaCl_2$ mixture the constricting power of 13 mgr. Ca-ions per Litre is so great, as to keep away œdema. In the presence of *more* than 13 mgr. Ca-ions e.g. 20 mgrs, as is the case in GUNZBURG's fluid, œdema will ensue, if not a certain amount of K is added to counteract their effect. A similar phenomenon appeared in the case of the kidneys viz. that an excess of Ca-ions content of the circulating fluid produced permeability of the glomerular epithelium for glucose [1]). The same thing was found by BRINKMAN [2]) with regard to the red blood corpuscles. Likewise NEUSCHLOSZ [3]) physico-chemical experiments demonstrated that the surface-tension of a lecithin-suspension in NaCl is as well influenced in the same manner by too little as by too much Ca.

We now tried to ascertain, whether œdema would arise when using a mixture of NaCl—$CaCl_2$ which contained much more than 0,007 % of $CaCl_2$. 6 aq., just as had occurred when the amount was 0,006 %.

With a view to this **0.01** % $CaCl_2$ 6 aq. (instead of 0,007 %) was dissolved in NaCl 0.6 %. We expected œdema to come forth. It did not come forth, though, which was owing to a quite unexpected phenomenon: *the perfusion of the liquid through the vessels stopped abruptly.* It could not be restored even through a considerable rise of the hydrostatic pressure. When the same result was obtained several times running, also with higher Ca-ions concentrations [4]), further experiments concerning the influence of a higher Ca-ions concentration on the producing of œdema had to be relinquished, and vascular contraction was supposed to come into play.

Now the assumption was warrantable that the vascular contraction (spasm, tonus) would disappear on addition of K. It appeared, indeed *that when to the circulating fluid (NaCl* 0,6 % + *$CaCl_2$*. 6 *aq.* 0.01 %) 0.01 % *KCl was added the perfusion of the liquid was restored again.*

We are justified in concluding from this that when, in a system Na + Ca, the Ca-ions concentration is higher than agrees with $CaCl_2$. 6 aq. 0.007 %, a constricting influence is exercised on the vessels, which may be counteracted by K-ions. This action seems to be reversible; it may be repeated several times.

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[1]) HAMBURGER u. BRINKMAN, Biochem. Zeitschr. **95**, 101, 1919.

[2]) R. BRINKMAN, Broch. Zeitschr. **95**, 101 (1919).

[3]) NEUSCHLOSZ, Pflüger's Archiv f. d. ges. Physiol. **181**, 17, 1920.

[4]) More about this in a subsequent publication.

In correlating this result with the well-known observations of CHIARI and JANUSCHKE [1]), according to which the process of conjunctival inflammation may be arrested by instillation of a $CaCl_2$-solution, we are led to suppose that a definite concentration of Ca-ions exercises a constricting influence upon the vessels and at the same time a coagulating action upon the vascular wall. It appears then that both these actions are neutralised by K.

That the contracting and the coagulating action may coincide, is substantiated by the observations on the influence of oxygen. SÉVERINI [2]) found that oxygen brings about contraction of the vessels; while GUNZBURG [3]), reports that a potassium-free RINGER's mixture, which in other cases always caused œdema, did not cause it when the mixture was saturated with oxygen. From this it seems probable that *vascular contraction coincides with decrease of permeability of the vessel-wall* not only with a definite concentration of Ca-ions, but also under the influence of oxygen.

As regards the inhibition of vascular constriction through the addition of KCl to the mixture NaCl 0,6% + $CaCl_2$. 6 aq. 0.01%, it appears that the minimal required concentration of KCl is about KCl 0,004 %.

SUMMARY.

The described researches, of which a detailed report appeared in the Biochemische Zeitschrift [4]), may be summarized as follows:

1. When perfusing the frog's leg with an aqueous solution of NaCl and $CaCl_2$, potassium may be absent in the circulating fluid, without œdema being evoked. The circulating fluid, however, should contain a definite concentration of calcium-ions. NaCl 0.6% $CaCl_2$. 6 aq. **0.007%** will serve our purpose here. When using $CaCl_2$. 6 aq. **0.006 %** œdema will arise. This will occur also when to the first-named mixture 0.01 % KCl is added. This phenomenon finds an explanation in the fact that the coagulating action of the Ca-ions is counteracted by the antagonistic K-ions.

2. That GUNZBURG wanted potassium in his solution to prevent oedema is to be ascribed to the fact that he used an excess of calcium-ions. The arrest of œdema in GUNZBURG's experiments is,

[1]) CHIARI u. JANUSCHKE, Arch. f. exper. Pathol. u. Pharmakol. **65**, 120/126, 1911.

[2]) LUIGI SEVÉRINI, „Ricerche sulla innervazione dei vasi sanguigni". Perugia Boncompagni et Cie (See BAYLISS: „Principles of General Physiology", 1915, p. 534.

[3]) GUNZBURG, l.c.

[4]) RUDOLF J. HAMBURGER, Bioch. Zeitschr. **129**, 153, 1922.

therefore, *not owing to a specifically radio-active potassium-action* but only to the long known potassium-calcium antagonism.

3. The influence of the K-ion concentration upon the permeability of the vessel-wall (mentioned sub 1 and 2) coincides with an influence upon the capillary lumen. A perfusion of the vascular system of the frog with a mixture of NaCl 0.6 % + $CaCl_2$. 6 aq 0.01 % produces so considerable a constriction of the vessels that no fluid can pass through any more. *When to this mixture a little KCl, say,* 0.01 % *KCl is added, the vessels dilate and the fluid runs on as before. This process is reversible.*

4. The parallelism of decrease of permeability and of constriction of the vessel-wall manifests itself not only under the influence of Ca-ions, but also under that of oxygen.

5. In a quantitative determination of the dilating and constricting action of pharmaca after TRENDELENBURG, due regard should be paid in future to the ratio of Potassium- and Calcium-ions in the circulating fluid.

From the Physiological Laboratory of the
5 March 1922. *University of Groningen.*

ERRATUM.

In these Proceedings of June **1912** (Vol. XV), p. 276, Table II, column 5, line 12 from the bottom to replace the there printed number 1.18908 by the number **1.69487.**